Advanced Calculus (MAA 4212) Due Friday 1/25

Homework #2

$\boxed{1}$ Let $f_n : (-1,1) \to \mathbb{R}$ be given by x^n for all $n \geq 0$. Prove that

(a) the sequence (f_n) converges pointwise (and determine the function it converges to) and

(b) the sequence (f_n) does not converge uniformly.

$\boxed{2}$ Suppose that the function $f : [0,1] \to \mathbb{R}$ is continuous and define the functions $g_n : [0,1] \to \mathbb{R}$ by $g_n(x) = x^n f(x)$. Prove that

(a) if the sequence (g_n) converges uniformly then $f(1) = 0$ and

(b) if $f(1) = 0$ then the sequence (g_n) converges uniformly.