Homework 4

1. Show that the function $f : [0, 1] \to \mathbb{R}$ defined by $f(x) = x^2$ is Riemann integrable. Your proof should use Lemma 11.11 and the identity

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$ 

2. Suppose $f : [-1, 1] \to \mathbb{R}$ takes nonnegative values. Show that if $f$ is Riemann integrable and continuous at 0 and $f(0) > 0$ then

$$\int_{-1}^{1} f \, dx > 0.$$