Homework 5

$\boxed{1}$ Prove that if $f : [0,1] \to \mathbb{R}$ is bounded and the lower integral of f is positive then there is an open interval on which $f > 0$.

$\boxed{2}$ Define the functions $f_n : [0,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} n & \text{if } 0 < x \leq 1/n, \text{ and} \\ 0 & \text{otherwise (if } x = 0 \text{ or } x > 1/n). \end{cases}$$

Explain why each f_n is Reimann integrable, the sequence (f_n) converges pointwise to a Riemann integrable function, but

$$\lim_{n\to\infty} \int_0^1 f_n \, dx \neq \int_0^1 f \, dx,$$

even though the limit on the left hand side exists.