

Homework 8

1 We have defined what it means for two norms to be equivalent. Prove that this relation really is an equivalence relation, i.e., that it is symmetric, reflexive, and transitive.

2 Consider the vector space $C([0, 1])$ with the norms

$$\|f\|_2 = \sqrt{\int_0^1 |f|^2 dt}$$

and

$$\|f\|_\infty = \max\{|f(t)| : 0 \leq t \leq 1\}.$$

Let $f_n(t) = t^n$ (defined on $[0, 1]$) and show that

$$\lim_{n \rightarrow \infty} \|f_n\|_2 = 0$$

whereas $\|f_n\|_\infty = 1$ for all n . Conclude that the norms $\|\cdot\|_2$ and $\|\cdot\|_\infty$ are not equivalent.