Homework #1

Answers should be submitted on a separate piece of paper. You need not attach this page, although if you do, then you will have the questions when your work is returned. Work which is sloppy or messy or that which is not written in a clear and coherent fashion will be marked down.

1 Sketch (in the complex plane) the set of points determined by each of the following conditions. *No reasoning is required, just the sketches.*

(a) $|z - 1 + i| = 1$.

(b) $|z + 2 - i| \leq 3$.

(c) $|z - 1 - 4i| \geq 4$.

2 Let w and z denote the complex numbers

$$w = a + ib \quad \text{and} \quad z = c + id.$$

(a) Use algebraic manipulation to show that

$$|wz| = \sqrt{(a^2 + b^2)(c^2 + d^2)}.$$

(b) Explain how part (a) shows that

$$|wz| = |w||z|$$

for all complex numbers w and z.

3 Use part (b) of the previous exercise to show that

$$|z^n| = |z|^n$$

for all natural numbers $n \geq 0$. *Notes: for a formal proof, you may want to employ mathematical induction; by define* $z^0 = 1$ *for all complex numbers.*

4 Prove that

(a) the complex number z is real if and only $\overline{z} = z$, and

(b) the complex number z is either real or purely imaginary if and only if $\overline{z}^2 = z^2$.