Homework #2

1 Let S denote the sector in the complex plane defined by $r \leq 1$ and $0 \leq \theta \leq \pi/4$. Sketch the regions onto which S is mapped by the following functions. *No reasoning is required, just the sketches.*

(a) $f(z) = z^2$.

(b) $f(z) = z^3$.

(c) $f(z) = z^4$.

There are many theorems which make computing limits easier. However, when a question asks you to compute a limit *from the definition*, that means that to show that

$$\lim_{z \to z_0} f(z) = w_0,$$

you must show that for every positive real number $\varepsilon > 0$, there is another positive real number $\delta > 0$ such that

$$|z - z_0| < \delta \quad \text{implies} \quad |f(z) - w_0| < \varepsilon.$$

2 Using only the definition, show that $\lim_{z \to z_0} \operatorname{Re} z = \operatorname{Re} z_0$.

3 Let a and b denote arbitrary complex constants. Using only the definition, show that $\lim_{z \to z_0} (az + b) = az_0 + b$.