Multiple Choice Questions

1. Find \( \frac{dy}{dx} \) if \( y = \sin(\tan(3x)) \)

(a) \( \frac{dy}{dx} = 3 \cos(3x) \sec^2(3x) \)

(b) \( \frac{dy}{dx} = 3 \cos(\sec^3(3x)) + \tan(3x) \cos(3x) \)

(c) \( \frac{dy}{dx} = \cos(\tan(3x)) \sec(3x) \tan(3x) \)

(d) \( \frac{dy}{dx} = 3 \cos(\tan(3x)) \sec^2(3x) \)

(e) \( \frac{dy}{dx} = 3 \cos(\sec^2(3x)) \)

2. Find \( f'(1) \) if \( f(x) = \frac{2x^\pi}{\pi} - 4^{x^2+1} \).

(a) \( 2 - 32 \ln 4 \)

(b) \( \frac{2}{\pi} - 2 \ln 4 \)

(c) \(-6 \)

(d) \( \frac{2}{\pi} - 16 \)

(e) \( 2 - 16 \ln 4 \)

3. If an oscillating sprinkler makes an angle of \( \theta \) radians with the ground, the range is covered (horizontal distance in feet) is given by the formula \( R = \frac{1}{2} \sin \theta \cos \theta \). At what rate the range \( R \) is changing with respect to \( \theta \) when \( \theta = \frac{\pi}{6} \)

(a) The range is decreasing by 0.25 feet per radian.

(b) The range is decreasing by 1 foot per radian.

(c) The range is increasing by 0.25 feet per radian.

(d) The range is decreasing by \( \sqrt{3}/8 \) feet per radian.

(e) The range is increasing by \( \sqrt{3}/8 \) feet per radian.
4. Write the equation of the normal line to the graph of the function \( f(x) = \frac{x^{4/3} + 2e^x}{x} \) at \( x = 1 \).

   (a) \( y = 3x + (4 + 2e) \)
   (b) \( y = 3x - (4 + 2e) \)
   (c) \( y = -3x + (4 - 2e) \)
   (d) \( y = -3x - (4 - 2e) \)
   (e) \( y = -3x + (4 + 2e) \)

5. At which value(s) of \( x \) does the function \( g(x) = x\sqrt{4x + 3} \) have a horizontal tangent line?

   (a) \( x = -1 \) and \( x = -5/6 \)
   (b) \( x = -5/4 \) only
   (c) \( x = -1 \) only
   (d) \( x = -5/6 \) only
   (e) No horizontal tangent lines

6. Find the slope of the tangent line to \( x^3 + y^2 + xy + e^{x-1} = 8 \) at the point \((1, -3)\)

   (a) \(-7/5\)
   (b) 2
   (c) 4/5
   (d) 3/8
   (e) 1/5

7. Let \( h(x) = [xf(x) - 4]^2 \). Find \( h'(2) \) if \( f(2) = 1/2 \) and \( f'(2) = -1 \).

   (a) 9
   (b) -6
   (c) 6
   (d) -5
   (e) 15
8. Find the slope of the normal line to the graph of \( f(x) = \sec^2(x) \) at \( x = \frac{\pi}{4} \).

(a) \( -\frac{1}{4} \)
(b) \( \frac{1}{4} \)
(c) \( -4 \)
(d) \( 4 \)
(e) \( -\frac{1}{8} \)

9. If \( g(x) = \tan^{-1}(e^x) \), find \( g''(x) \).

(a) \( g''(x) = \frac{e^x}{1 + e^{2x}} \)
(b) \( g''(x) = \frac{2e^{2x}}{(1 + e^{2x})^2} \)
(c) \( g''(x) = \frac{e^x - e^{-3x}}{(1 + e^{2x})^2} \)
(d) \( g''(x) = \frac{2e^{2x} + e^x}{(1 + e^x)^2} \)
(e) \( g''(x) = \frac{-e^{2x}}{(1 + e^{2x})^2} \)

10. A cylinder is being flattened so that its volume does not change. Find the rate of change of radius when \( r = 3 \) inches and \( h = 4 \) inches if the height is decreasing at 0.2 in/sec.

(a) \( \frac{3}{20} \) in/sec
(b) \( \frac{3}{40} \) in/sec
(c) \( \frac{2}{15} \) in/sec
(d) \( -\frac{3}{20} \) in/sec
(e) \( -\frac{2}{15} \) in/sec

11. The dollar cost of producing \( x \) bagels is \( C(x) = 300 + 6.25x - \frac{0.5}{1000^2} x^3 \). Determine the marginal cost in dollars per unit of producing 2000 bagels.

(a) \( 3.25 \)
(b) \( -0.25 \)
(c) \( 0.25 \)
(d) \( 2.5 \)
(e) \( -3.25 \)
12. Use linearization for a suitable function at a suitable point to approximate \( \frac{1}{\sqrt{0.8}} \).

(a) 1.2
(b) 1.05
(c) 0.9
(d) 1.01
(e) 1.1

13. Find each \( x \)-value on \([0, 2\pi]\) at which the tangent line to \( f(x) = 3x - 2\cos x \) is parallel to the line \( 4x - 2y = 6 \).

(a) \( x = \pi/2 \) and \( 3\pi/2 \)
(b) \( x = 4\pi/3 \) and \( 5\pi/3 \)
(c) \( x = \pi/6 \) and \( 5\pi/6 \)
(d) \( x = \pi/3 \) and \( 5\pi/3 \)
(e) \( x = 7\pi/6 \) and \( 11\pi/6 \)

14. Use the definition of the derivative to evaluate the limit

\[
\lim_{x \to \ln 2} \frac{e^{2x} - 4}{x - \ln 2}
\]

(a) 2
(b) 4
(c) 8
(d) 0
(e) 1
Free Response Questions

1. A ladder 13 meters long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 0.6 m/sec. How fast is the top sliding down the wall when the foot of the ladder is 5 meters from the wall?

2. Use logarithmic differentiation to find the slope of the tangent line to \( f(x) \) at \( x = 1 \) for

\[
f(x) = \frac{e^{x^2 - x} \sqrt{6 + 3x^2}}{\tan^{-1}(x)(3x - 1)^{4/3}}
\]
3. A particle is moving along a number line so that its position $s$ (in meters) after $t$ minutes is given by the function $s(t) = 16t^3 + 12t^2 - 144t$ for $t \geq 0$.

(a) Write formulas for the velocity and the acceleration of the particle at every time $t$.

(b) On what open interval is the particle travelling in the negative direction?

(c) Find the total distance travelled by the particle in the time interval $[0, 2]$.

(d) On what interval is the particle is slowing down?
4. Find the value $a$ if the tangent line to $f(x) = \frac{1}{\sqrt{x} + 1}$ at $x = a$ passes through the point $(1, 0)$.
Answer Key

Multiple Choice Questions:

Free Response Questions:
1. -4 m/sec
2. \(-\frac{8}{9} - \frac{2}{\pi}\) \(\frac{4}{\pi} \left(\frac{9}{16}\right)^{1/3}\)
3. (a) \(v(t) = 48t^2 + 24t - 144\) and \(a(t) = 96t + 24\)  
   (b) on (0, 3/2)  
   (c) 738 meters (I hope this is correct (sigh!))  
   (d) on (0, 3/2)  
4. \(a = -1/3\)