3. (Version of #17 of L15 HW) Find the equation of the normal line to the graph of \( y = \sec^2(x) \) at \( x = \frac{\pi}{4} \).

We need:  
1. **Slope of the Normal Line**

\[
\left( = \frac{-1}{\text{slope of the tangent line at } x = \frac{\pi}{4}} \right)
\]

\[
\left( = \frac{-1}{f'(\frac{\pi}{4})} \right)
\]

2. **Point**

\[
\frac{\sec^2 \frac{\pi}{4}}{\cos^2 \frac{\pi}{4}} = \frac{1}{(\frac{\sqrt{2}}{2})^2} = 2
\]

\[
\left( = \frac{1}{\cos^2 \frac{\pi}{4}} \right)
\]

\[
\left( = \frac{1}{(\frac{\sqrt{2}}{2})^2} \right)
\]

\[
\left( = 2 \right)
\]

\[
f(x) = y = \sec^2 x \quad \text{(Use Power rule with the chain rule)}
\]

\[
f'(x) = \frac{dy}{dx} = 2 \cdot \sec x \cdot \frac{d}{dx} (\sec x \cdot \tan x)
\]

\[
= 2 \cdot \sec^2 x \cdot \tan x
\]

\[
w = f'(\frac{\pi}{4}) = 2 \cdot \sec^2 \left( \frac{\pi}{4} \right) \cdot \tan \left( \frac{\pi}{4} \right)
\]

\[
= 2 \cdot 2 \cdot 1
\]

\[
= 4
\]

1. \[
\frac{w}{w_{\text{Normal}}} = -\frac{1}{4}
\]

2. \[
\left( \frac{\pi}{4}, f \left( \frac{\pi}{4} \right) \right) = \left( \frac{\pi}{4}, \sec^2 \frac{\pi}{4} \right) = \left( \frac{\pi}{4}, 2 \right)
\]

**Eq. of the normal line:**

\[
y - 2 = -\frac{1}{4} \left( x - \frac{\pi}{4} \right)
\]

\[
y = -\frac{x}{4} + \frac{\pi}{16} + 2
\]

**University of Florida Honor Code:** On my honor, I have neither given nor received unauthorized aid in doing this assignment.

.................................

Signature
1. Find the value(s) of $x$ at which the graph of $y = \frac{3x}{(4x+1)^{2/3}}$ has a horizontal tangent line.
   
   (a) $x = -\frac{3}{10}$ and $x = -\frac{1}{4}$
   (b) $x = -\frac{3}{10}$ only
   (c) $x = -\frac{3}{4}$ and $x = -\frac{1}{4}$
   (d) $x = -\frac{3}{4}$ only
   (e) There is no horizontal tangent line.

   \[
   y' = \frac{3x}{(4x+1)^{2/3}} \rightarrow \text{use Quotient Rule.}
   \]

   \[
   \frac{dy}{dx} = \frac{(4x+1)^{-2/3} \cdot 3 - \frac{3x}{3} \cdot 4}{(4x+1)^{2/3}}
   \]

   \[
   = \frac{3(4x+1) - 8x}{(4x+1)^{4/3}} \cdot \frac{1}{(4x+1)^{2/3}}
   \]

   \[
   = \frac{3}{(4x+1)^{4/3}}
   \]

   \[
   \frac{dy}{dx} = \frac{4x+3}{(4x+1)^{5/3}}
   \]

2. Let $F(x) = f(g(x^3))$. Find $F'(1)$ if

   \[
   f(-1) = 0 \quad f(1) = -2 \quad g(1) = -1
   \]

   \[
   f'(-1) = 3 \quad f'(1) = 4 \quad g'(1) = \frac{1}{2}
   \]

   (a) 1
   (b) 2
   (c) 3
   (d) $\frac{9}{2}$
   (e) $\frac{3}{2}$

   \[
   f'(x) = f'(g(x^3)) \cdot \frac{d}{dx} (g(x^3)) \quad \text{(Chain Rule)}
   \]

   \[
   F'(x) = f'(g(x^3)) \cdot g'(x^3) \cdot 3x^2 \quad \text{(Chain Rule)}
   \]

   \[
   F'(1) = f'(g(1)) \cdot g'(1) \cdot 3
   \]

   \[
   = f'(-1) \cdot \frac{1}{2} \cdot 3 = 3 \cdot \frac{1}{2} \cdot 3 = \frac{9}{2}
   \]