1. Express the following limit of a Riemann sum as a definite integral:

\[ \lim_{n \to \infty} \sum_{i=1}^{n} \left( 1 + \frac{3i}{n} \right)^{\frac{3}{n}} \Delta x \]

Interval corresponding to each integral

(a) \( \int_{0}^{3} x^5 \, dx \)
(b) \( \int_{0}^{4} (1 + x)^5 \, dx \)
(c) \( \int_{1}^{4} (1 + x)^5 \, dx \)
(d) \( \int_{1}^{4} x^5 \, dx \)
(e) \( \int_{0}^{3} (1 + 3x)^5 \, dx \)

\[ \Delta x = \frac{3}{n} = \frac{b-a}{n} \]

Choose: \( \left[ 0, \frac{3}{n} \right] \) : Now \( x_i = a + i \Delta x = \frac{3i}{n} \).

\[ f(x_i) = (1 + \frac{3i}{n})^5 \]

So, \( f(x_i) = \left( 1 + \frac{3i}{n} \right)^5 \)

\( f(x) = (1 + x)^5 \)

Therefore:

\( \int_{0}^{3} (1 + x)^5 \, dx \)

2. Mr. Jones was driving along a rural highway at a speed of 60mph (88ft/sec) when he saw a deer on the roadway ahead. He applied the brakes with a constant deceleration of 22ft/sec². If \( t = 0 \) represents the time at which the brakes are first applied, find the distance traveled before the car comes to a stop.

Break applied

\( t = 0 \)
\( v(0) = 88 \)
\( s(0) = 0 \)

\( a(t) = -22 \)
\( v(t) = -22t + C_1 \)
\( v(0) = 88 \)

\( 88 = -22(0) + C_1 \)

\( C_1 = 88 \)

\( v(t) = -22t + 88 \)
\( s(t) = -11t^2 + 88t + C_2 \)
\( s(0) = 0 \)
\( 0 = -11(0)^2 + 88(0) + C_2 \)

\( C_2 = 0 \)

\( s(t) = -11t^2 + 88t \)

When car stops: \( v(t) = 0 \)
\( -22t + 88 = 0 \)
\( t = 4 \)

Car stops after 4 seconds. So the distance traveled is

\( s(4) = -11(4)^2 + 88(4) = 176 \)
3. (a) Find a Riemann sum which approximates the area under the graph of \( f(x) = 3x + 2 \) on \([0, 3]\) using \( n \) sub-intervals of equal width letting \( x_i^* \) be the right end point of the sub-interval \([x_{i-1}, x_i]\).

i. \( \Delta x = \frac{b-a}{n} \) ........................

ii. \( x_i = a + \frac{3i}{n} \) ........................

iii. \( R_n = \sum_{i=1}^{n} \left[ 3 \left( \frac{3i}{n} \right) + 2 \right] \frac{3}{n} = \frac{3}{n} \sum_{i=1}^{n} \left( \frac{9}{n} \right) i + \frac{6}{n} \)

(b) Find the exact area under the graph of \( f(x) = 3x + 2 \) on \([0, 3]\) by taking the limit of the Riemann sum as \( n \to \infty \).

Note: \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \), \( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \), \( \sum_{i=1}^{n} i^3 = \left[ \frac{n(n+1)}{2} \right]^2 \)

\[
\text{Area} = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{9}{n} \right) i + \sum_{i=1}^{n} \left( \frac{6}{n} \right)
\]

\[
= \lim_{n \to \infty} \frac{27}{n^2} \sum_{i=1}^{n} i + \frac{6}{n} \sum_{i=1}^{n} 1
\]

\[
= \lim_{n \to \infty} \frac{27}{n^2} \cdot \frac{n(n+1)}{2} + \frac{6}{n} \cdot n
\]

\[
= \frac{27}{2} + 6
\]

\[
= \frac{39}{2}
\]

Sorry, there is a typo in part (b). I wanted the interval to be \([0, 3]\). But what I had was \([0, 1]\).

If you do this on \([0, 2]\), Area = 10.