Exam 1.

Due Wednesday Oct. 7 before class.

1. Let a_n for $n \in \omega$ be finite sets, each of size at least two, with discrete topology. Let X be the product $\prod_n a_n$ with the product topology. Prove that X is homeomorphic to the Cantor space.
2. Let X_0 be a compact space, $X_1 \subset X_0$ a closed subset, and Y a Polish space. Show that the function $\pi\colon C(X_0, Y) \to C(X_1, Y)$ defined by $\pi(f) = f \restriction X_1$ is a continuous function. Prove that π-images of open sets are open if $Y = \mathbb{R}$. (*Hint.* For the second sentence, use either Tietze extension theorem or Urysohn lemma, textbook 1.2 or 1.3 on page 4.)
3. Let X be a Polish space. Show that the map $\pi\colon K(X) \times K(X) \to K(X)$ defined by $\pi(K, L) = K \cup L$ is a continuous function.
4. Let X be a Polish space. Show that the set $\{K \in K(X)\colon K$ has no isolated points$\}$ is a G_δ-subset of $K(X)$.