Exam 1.

Due Wednesday Nov. 21 (before Thanksgiving).

1. Let $X = \mathbb{R}$. Show that the set $\{K \in K(X) \colon K$ has an accumulation point$\}$ is Borel.

2. Let $X = \mathcal{P}(\mathbb{Q})$ with the usual topology (what is it?). Show in detail that the set $\{a \subset \mathbb{Q} \colon a$ contains an infinite decreasing sequence$\} \subset \mathcal{P}(\mathbb{Q})$ is analytic.

3. Show that there is no universal Borel set on ω^ω. That is, there is no Borel set $B \subset \omega^\omega \times \omega^\omega$ such that every Borel subset of ω^ω is a vertical section of ω^ω.

4. Let X be a Polish space. Let Γ be an analytic graph on X. Say that sets $A_0, A_1 \subset X$ are disconnected if there is no Γ-edge $\langle x_0, x_1 \rangle$ such that $x_0 \in A_0$ and $x_1 \in A_1$. Show that for any two disconnected, disjoint analytic sets $A_0, A_1 \subset X$ there are Borel, disconnected, and disjoint sets B_0, B_1 such that $A_0 \subseteq B_0$ and $A_1 \subseteq B_1$.