THE BANACH-SAKS RANK OF A WEAKLY COMPACT SET

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Abstract: A classical result by Mazur states that every weakly convergent sequence $(x_n)_n$ on a Banach space X has a norm-convergent convex-block subsequence $(y_n)_n$, $y_n = \sum_{m\in s_n} a_m x_m$. With this basic fact in mind, we will introduce the Banach-Saks rank of a separable weakly-compact subset A of the space X, that measures the complexity of the appropriate convex-combinations $(a_m)_{m\in s_n}$ needed to guarantee the norm-convergence. For example, when the combinations are the trivial ones, i.e. $s_n = \{m_n\}$ and $a_{m_n} = 1$, then we are saying that the subsequence $(x_{m_n})_n$ converges in norm; and if this is the case for every sequence in A, then we are saying that A is in fact a (norm-)compact set. In for example the convex combinations needed for each sequence in A are longer and longer averages, i.e., $a_m = 1/|s_n|$ for every $m \in s_n$ and $|s_m| \to_m \infty$, then we are reflecting the fact that A has the *Banach-Saks* property, and so on.

We expect the talk to be (quite) self contained, with a minimal amount of Banach space theory and mostly with combinatorial and (some) set-theoretical arguments.

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