SEALS 2018 ABSTRACTS

Speaker: Francis Adams

Title: The Loose Number of Topologically Presented Graphs

Abstract: Given a graph G on a topological space X, we define the loose number of G, a cardinal invariant which depends on both the graph-theoretic properties of G as well as the topology on the vertex set X. In the setting of separable metric spaces, we can relate the loose number to two well known graph invariants: the chromatic number and the coloring number. Evaluating this cardinal leads to interesting connections with forcing, infinitary combinatorics, descriptive set theory, and topology. We discuss these connections and provide many examples. Much of this work is joint with Jindrich Zapletal.

Speaker: Rachael Alvir

Title: Scott ranks of scattered linear orders

Abstract: The logic $L_{\omega 1 \omega}$ is obtained by closing finitary first-order logic under countable disjunction and conjunction. There is a kind of normal form for such sentences. The Scott rank of a countable structure A is the least complexity of a sentence Φ_A of $L_{\omega 1 \omega}$ which describes A up to isomorphism among countable structures. We calculate the Scott ranks of all countable scattered linear orders, along the way calculating the back-and-forth relations on this class. This result generalizes those previously obtained results on the Scott ranks of ordinals, and attempts to partially answer an open question of Chris Ash's from 1987, about the closure properties of the class of linear orders with α-friendly copies under finite sum and product.

This is joint work with Dino Rosseger (TU Vienna).

Speaker: George Barmpalias

Title: From algorithmic learning of languages to learning probability distributions (and back)

Abstract: Algorithmic learning theory traditionally studies the learnability of grammars given sufficiently long texts, while recent work by (Vitanyi and Chater 2017) and (Bienvenu et al. 2014) has adapted this framework to the study of learnability of probability distributions from random data. In this study, one is given a sufficiently long stream of random data and the task is to guess a probability distribution with respect to which the data is algorithmically random. We show certain equivalences between algorithmic learning of languages and probability distributions, that allow to transfer much of the classic theory to the study of algorithmic learning of probability distributions. In particular, we prove that for certain families of probability measures that are parametrized by reals (texts), learnability of a subclass of probability measures is equivalent to learnability of the class of the corresponding real parameters. Based on these equivalences, we present a number of applications, providing many new results regarding explanatory and behaviorally correct learnability of classes of measures, thus drawing parallels between the two learning theories.

This is joint work with Nan Fang and Frank Stephan.

Speaker: Dana Bartosova

Title: On a problem of Ellis

Abstract: We study an old question of Ellis whether two distinguished flows of a topological group are canonically isomorphic. We present a negative answer for some automorphism groups of countable structures and ask set-theoretic questions that the analysis of these flows leads us to. This is a joint work with Andy Zucker.

Speaker: Anton Bernshteyn

Title: The Lovasz Local Lemma and measurable graph colorings

Abstract: A natural direction of inquiry in descriptive combinatorics is studying the extent to which the known techniques and methods developed in finite combinatorics can be adapted for the measurable setting. A particularly useful family of combinatorial tools is collectively referred to as the *probabilistic method*. This includes the so-called *Lovasz Local Lemma* (the *LLL* for short), which is indispensable for working with graph colorings. I will talk about measurable versions of the LLL and some of their consequences in measurable combinatorics, as well as about cases when the LLL fails in the measurable setting.

Speaker: Ronnie Chen

Title: Structurability by contractible simplicial complexes

Abstract: In this talk, we will discuss the classes of countable Borel equivalence relations which are structurable by *n*-dimensional contractible simplicial complexes, for n = 1, 2,…, ∞. The case n = 1 is the well-studied class of treeable equivalence relations. Generalizing a classical result of Jackson-Kechris-Louveau in the treeable case, we show that for each $n<\infty$, there is a constant $M_n<\infty$ such that every equivalence relation structurable by n-dimensional contractible simplicial complexes Borel embeds into an equivalence relation structurable by such complexes with the further property that each vertex belongs to at most M_n edges. Our proof also yields that (for the case $n=\infty$) every countable Borel equivalence relation Borel embeds into an equivalence relation structurable by locally finite contractible simplicial complexes.

Speaker: Peter Cholak

Title: Encodable by thin sets

Abstract: Let c be a coloring of n-tuples of ω by finitely many colors. For I less than the number of colors, a set T is *I-thin* iff c uses at most I colors to color all the n-tuples from T. The statement such a thin set exists is called $RT^n_{<\infty, l}$.

We say a set S is $RT^n_{<\infty, l}$ -*encodable* iff there is a coloring c as above such that every I-thin set computes S. Wang showed that when I is ``large'' only the computable sets are $RT^n_{<\infty, l}$ -encodable. Dorais, Dzhafarov, Hirst, Mileti, and Shafer showed that the hyperarithmetic sets are $RT^n_{<\infty, l}$ -encodable for ``small'' I. Cholak and Patey showed that the arithmetic sets are $RT^n_{<\infty, l}$ -encodable for ``medium'' I. Of course, what is missing here is the exact definition of small, medium, and large. In the talk we will provide ``tight'' definitions, at least, for a ``few'' n. This is joint work with Ludovic Patey.

Speaker: Su Gao

Title: The graph homomorphism problem for the Bernoulli shift on Z^2

Abstract: Let $F(2^{Z2})$ be the Cayley graph on the free part of the Bernoulli shift on 2^{Z2}. The graph homomorphism problem for $F(2^{Z2})$ asks for which finite graphs ΓΓ there exists a continuous or Borel graph homomorphism from $F(2^{Z2})$ to Γ. In this talk we report some results on the continuous version of the graph homomorphism problem. We first demonstrate a theorem known as the Twelve Tiles Theorem that completely characterizes the positive answers to the continuous graph homomorphism problem. Then we show that the set of (codes for) all finite graphs Γ such that there exist continuous graph homomorphisms from $F(2^{Z2})$ to Γ is a Σ^0_1-complete set. This is joint with Steve Jackson, Ed Krohne, and Brandon Seward.

Speaker: John Clemens

Title: Borel partition properties for equivalence relations

Abstract: One may consider definable analogues of partition relations in Ramsey theory, where we consider Borel partitions of tuples from a quotient spaces X/E, for X a Polish space and E a Borel equivalence relation on X. The size of a homogeneous set for a partition may then be measured in terms of Borel reducibility of equivalence relations. In this talk I will discuss this viewpoint, and focus on partition relations related to the equivalence relation F_2 of equality of countable sets of reals. A result of Kanovei-Sabok-Zapletal may be interpreted in this context to give a version of the Pigeonhole Principle: For any Borel partition of F_2-classes, there is a set of the same definable cardinality as F_2 which is either homogeneous or discrete for the partition. On the other hand, I will show that a natural strengthening of this result, a definable version of weak compactness, fails: There is a Borel 2-coloring of pairs of F_2 classes for which there is no homogeneous set with definable cardinality of F_2.

Speaker: Valentina Harizanov

Title: Effective products of computable structures

Abstract: We consider an effective product of computable structures where elements are partial computable functions and a cohesive set is used to determine truth of formulas. A cohesive set is an infinite set that is indecomposable with respect to computably enumerable sets. In particular, a maximal set is a computably enumerable set with a cohesive complement. We investigate which first-order formulas are true in an effective product. For some familiar structures, we analyze the isomorphism types of their effective powers, and show how these powers arise naturally in computable algebra. Recent work is joint with Rumen Dimitrov.

Speaker: Julia F. Knight

Title: Intrinsic computing power of structures

Abstract: Most of the time, people in computability work with subsets of ω, and in computable structure theory, we usually limit ourselves to countable structures for a computable language. If the universe is a subset of ω, we identify the structure with its atomic diagram, which we code by a subset of ω. Using *Muchnik reducibility*, we can compare the ``intrinsic'' computing power of countable structures. We write $A \leq_w B$ if every copy of B (with universe a set of natural numbers) computes a copy of A. Like other mathematicians, people in computable structure theory are interested in uncountable structures such as R (the ordered field of reals). Noah Schweber extended Muchnik reducibility in a way that lets us compare the intrinsic computing power of structures of arbitrary cardinality. We write $A \leq_w^* B$ if, after a collapse of cardinals that makes both structures countable, every copy of B (with universe a set of natural numbers) computes a copy of A. I will describe results (with various co-authors) using Schweber's notion of *generic Muchnik reducibility* to compare various structures related to R.

Speaker: Steffen Lempp

Title: The computational complexity of models of strongly minimal theories

Abstract: In the mid-1990's, I raised the question of what one can say about the possible computational complexity of all the countable models of a strongly minimal theory if one only knows that one model is computable. Based on prior work, especially recently by Andrews and Knight, we can now give a precise answer: The models can be computed precisely by all the degrees d high over 0'', i.e., d >= 0'' such that d'' >= 0''''. This is joint work with Andrews and Schweber.

Speaker: Andrew Marks

Title: A constructive solution to Tarski's circle squaring problem

Abstract: In 1925, Tarski posed the problem of whether a disc in R^2 can be partitioned into finitely many pieces which can be rearranged by isometries to form a square of the same area. Unlike the Banach-Tarski paradox in R^3, it can be shown that two Lebesgue measurable sets in R^2 cannot be equidecomposed by isometries unless they have the same measure. Hence, the disk and square must necessarily be of the same area.

In 1990, Laczkovich showed that Tarski's circle squaring problem has a positive answer using the axiom of choice. We give a completely constructive solution to the problem and describe an explicit (Borel) way to equidecompose a circle and a square. This answers a question of Wagon.

Our proof has three main ingredients. The first is work of Laczkovich in Diophantine approximation. The second is recent progress in a research program in descriptive set theory to understand how the complexity of a countable group is related to the Borel cardinality of the equivalence relations generated by its Borel actions. The third ingredient is ideas coming from the study of flows in networks.

This is joint work with Spencer Unger.

Speaker: Ethan McCarthy

Title: Cototal enumeration degrees and the Turing degree spectra of minimal subshifts

Abstract: A subset A of ω is cototal under enumeration reducibility if A is enumeration reducible to $2^{\omega}\backslash A$, that is, if the complement of A is total. We show that the e-degrees of cototal sets characterize the e-degrees of maximal anti-chain complements, the e-degrees of enumeration-pointed trees on $2^{<\omega}$, and the e-degrees of languages of minimal subshifts on 2^{ω}. Finally, we obtain a characterization of the Turing degree spectra of nontrivial minimal subshifts: they are the enumeration cones of cototal sets.

Speaker: Yann Pequignot

Title: Embeddability on functions: order and chaos

Abstract: We study the quasi-order of topological embeddability on continuous functions between Polish zero dimensional spaces. Our main result is the following dichotomy: the embeddability quasi-order on continuous functions from a given compact space to another is either an analytic complete quasi-order or a well-quasi-order.

We also investigate the existence of maximal elements with respect to embeddability in a given Baire class. We prove that no Baire class admits a maximal element, except for the class of continuous functions which admits a maximum element.

Speaker: Christopher Porter

Title: Aspects of Bernoulli randomness

Abstract: This project brings together two strands of research in algorithmic randomness: (1) randomness with respect to computable measures and (2) randomness with respect to random measures. In particular, we examine which sequences are random with respect to a Bernoulli measure with a parameter p that is itself random with respect to some computable measure.

As anticipated by work of Vovk and V'yugin, Freer and Roy, and Hoyrup, all such sequences are, in fact, random with respect to a computable measure; such measures are obtained by taking a mixture of a collection of random measures. We further investigate the extent to which randomness with respect to a non-computable Bernoulli measure is compatible with randomness with respect to some computable measure, as well as which random Bernoulli measures can be mixed to obtain a computable measure. This is joint work with Quinn Culver.

Speaker: Diego Rojas

Title: Online Computability and Differentiation in the Cantor Space

Abstract: This paper investigates a notion of differentiation for functions on the Cantor space. We study the existence and complexity of this derivative, particularly for online and computable online functions. It is shown that a random online function has no derivative at any computable point. It is shown that if a computable online function F has derivative m>0 at a weakly 1-random point, then F has derivative m on a set of positive measure. We also explore the family of online functions on the Cantor space which represent real-valued functions.

Speaker: Slawomir Solecki

Title: Fraïssé Limits and Compact Spaces

Abstract: Fraïssé theory is a method in classical Model Theory of producing canonical limits of certain families of finite structures. It turns out that this method can be dualized, with the dualization producing projective Fraïssé limits, and applied to the study of compact metric spaces. I will describe recent
results, due to several people, on connections between projective Fraïssé limits and the structure of some canonical compact spaces and their homeomorphism groups (the pseudoarc, the Menger curve, the Lelek fan, simplexes with the goal of developing a projective Fraïssé homology theory).

Speaker: Douglas Ulrich

Title: Borel complexity and Biembeddability Relations

Abstract: The following statement is independent of ZFC (assuming large cardinals): there is an absolutely Δ^1_2 reduction from graphs to colored trees that takes nonisomorphic graphs to nonbiembeddable colored trees. I discuss this and related results, including that it is consistent with ZFC that torsion-free abelian groups are maximal under absolute Δ^1_2-reducibility. Many of these results are joint with Shelah.

Speaker: Zoltan Vidnyanszky

Title: Borel chromatic numbers: basis and antibasis results

Abstract: The G_0 dichotomy states that there exists a Borel graph of uncountable Borel chromatic number that admits a Borel homomorphism to each such graph with uncountable Borel chromatic number. Hence, the collection of graphs with uncountable Borel chromatic number has a single element basis. We show that the analogous statement fails for the collection of graphs with infinite Borel chromatic numbers, in fact, there is no Borel graph of chromatic number at least 4 which would admit a homomorphism to each graph with infinite Borel chromatic number. Thus, the only remaining question is the existence of a single element basis for the collection of Borel graphs with chromatic number 3. We show that in this case the answer is affirmative.

Speaker: James Walsh

Title: Hierarchies of proof-theoretic strength

Abstract: It is well-known that natural axiomatic theories are well-ordered by proof-theoretic strength, for various popular metrics of proof-theoretic strength. However, without a precise mathematical definition of "natural," it is unclear how to prove this, or even how to state it. We will discuss two approaches to explaining this phenomenon, both of which emphasize the role of iterated reflection principles. The first approach is from joint work with Antonio Montalbán; the second approach is from joint work with Fedor Pakhomov.

Speaker: Linda Westrick

Title: Determined Borel sets in reverse math

Abstract: The standard definition of a Borel code in reverse math doesn’t require the model to believe that each real is either in the coded set or in its complement. In fact, the statement “for every Borel coded set, either it or its complement is non-empty” already implies ATR0. We define a determined Borel code to be a Borel code with the property that every real is contained either in the coded set or in its complement. While the statement “every Borel set has the property of Baire” is equivalent to ATR0 due to the above-mentioned technicality, the statement “every determined Borel set has the property of Baire” is not. We discuss the strength of this statement and other statements involving Borel sets in reverse math. Joint work with Astor, Dzhafarov, Montalban and Solomon.

Speaker: Guohua Wu

Title: A result towards Kierstead's conjecture for linear orders

Abstract: In this talk, I will present a recent work on Kierstead's conjecture for linear orders, generalizing the work of Cooper, Harris and Lee. In particular, we will show that Kierstead's conjecture is true for the order types $\Sigma\, \lim_{q\in Q} F(q)$, where F is an extended 0'-limitwise monotonic function (i.e., F can take value ζ). In contrast to Cooper, Harris, and Lee's work, the linear orders in our consideration can have finite and infinite blocks simultaneously. Our result also covers one case of Downey and Moses' work. It is a joint work with Maxim Zubkov from Kazan.