SEALS 2020 ABSTRACT BOOK

Rachael Alvir: *Scott Sentences and Finitely α-generated Structures*

Abstract: In this talk, we will define the notion of a finitely α-generated structure, and show some new results about their Scott sentences that generalize those known for finitely generated structures. We will show how these results can be used to the connect some of the existing non-equivalent definitions of Scott rank.

Kostas Beros: *Hypercyclic vectors and definable complexity*

Abstract: A *w*eight sequence $w \in \{1, 2\}^\omega$ of ones and twos determines a unilateral weighted shift B_w on the Hilbert space ℓ^2. An element $y \in \ell^2$ is *hypercyclic* for w iff the forward iterates of y under B_w are dense in ℓ^2. It is immediate that the set $\mathrm{HC}(w)$ of hypercyclic vectors for w is a G_δ set. In this talk, I will discuss recent work which shows that by looking at the common hypercyclic vectors for a *s*et of weight sequences, this complexity can be increased arbitrarily. Moreover, under CH, there are sets of weight sequences whose common hypercylic vectors do not even have the Baire property. This is joint work with Paul Larson.

Ruiyuan Chen: *Imaginary sorts and strong conceptual completeness for $L_{\omega_1\omega}$*

Abstract: Given a theory T and a formula ϕ, we may uniformly assign a set to each model of T, namely the interpretation of ϕ in it; moreover, this assignment is equivariant with respect to isomorphisms between models. More generally, this can be done if ϕ is replaced with an imaginary sort over T. We show that conversely, any Borel, isomorphism-equivariant assignment of a countable set to every countable model of a countable $L_{\omega_1\omega}$-theory T is named by an $L_{\omega_1\omega}$-imaginary sort. This result is the main ingredient in a strong conceptual completeness theorem (in the sense of Makkai) for $L_{\omega_1\omega}$, allowing any countable $L_{\omega_1\omega}$-theory to be canonically recovered up to bi-interpretability from its Borel groupoid of countable models.

Tomasz Ciesla: *Completeness of the homeomorphism relation of locally connected continua*

Abstract: Consider orbit equivalence relations induced by actions of Polish groups on standard Borel spaces. We say that an orbit equivalence relation E on X is Borel reducible to an orbit equivalence relation F on Y if there exists a Borel map $f\colon X \to Y$ so that $xEy \iff f(x)Ff(y)$ for all $x, y \in X$. A relation E is complete if all orbit equivalence relations are Borel reducible to E. We prove that the homeomorphism relation of locally connected continua is complete.

Samuel Coskey: *Jumps and the classification of scattered linear orders*

Abstract: We begin by studying the classification problem for countable scattered linear orders. Letting $\cong_\alpha$ denote isomorphism of scattered orders of rank α, we can define the "$\mathbb{Z}$-jump" of equivalence relations in such a way that the $\mathbb{Z}$-jump of $\cong_\alpha$ is $\cong_{\alpha+1}$. More generally, for any countable group Γ, we will define the Γ-jump of equivalence relations. After introducing the basic theory of these jump operators, we will discuss the question of when the Γ-jump is proper. In particular we will show the $\mathbb{Z}$-jump is proper, and hence the complexity of $\cong_\alpha$ increases properly with α. This is joint work with John Clemens.

Michael Cotton: *Weiss problem for uncountable abelian Polish groups*

Abstract: "Weiss' problem" typically refers to the question of whether Borel actions of countable amenable groups on standard Borel spaces induce only hyperfinite equivalence relations. Gao and Jackson showed that this is true of countable abelian groups, and much progress has been made generalizing the result to wider classes of countable groups, though the question for all countable amenable groups remains wide open. In the case of uncountable Polish groups, it still makes sense to ask the question of which classes of groups induce only hyperfinite relations, and it has been conjectured that if an abelian orbit equivalence relation happens to be reducible to one with countable classes then it must reduce to hyperfinite. Ding and Gao showed this to be the case if the abelian group is also non-archimedean, and it has also been shown to be the case if the abelian group is locally compact. We will survey these results, discuss the locally compact case in a bit more detail, and also discuss some intermediate questions that may be of interest.

Trevor Davila: *Groups with infinite asymptotic dimension*

Abstract: Asymptotic dimension is a coarse geometric invariant introduced by Mikhail Gromov to study countable groups. Recently asymptotic dimension has provided new insights in the theory of countable group actions on Polish spaces. There are interesting groups with infinite asymptotic dimension, which has led to interest in weakenings of finite asymptotic dimension. In this talk we will survey these infinite dimensional properties, and conclude with a discussion of decomposition complexity growth. This is a dimension-like property introduced by me and is one of the weakest such properties that implies a coarse embedding into a Hilbert space.

Johanna Franklin: *Lowness for paths, isomorphism, and isometric isomorphism*

Abstract: We define lowness for paths in both Baire space and Cantor space and show that these two notions coincide not only with each other but with lowness for isomorphism and lowness for isometric isomorphism.

Su Gao: *Vershik's Conjecture, Ultraextensive Spaces, and the Herwig-Lascar Property of Groups*

Abstract: I will talk about some results from joint work with Mahmood Etedadialiabadi, Francois La Maitre, and Julien Melleray. We first verify a conjecture of Vershik which states that the isometry group of the universal Urysohn space (as well as the automorphism group of the random graph) contains a dense subgroup that is isomorphic to Hall's universal countable locally finite group. We then introduce a notion of ultraextensive metric spaces and state some properties of such spaces, including that their isometry groups all contain dense locally finite groups. Our work extends previous work by Bhattacharjee, McPherson, Vershik, Pestov, Solecki, and Rosendal. Most of these results can be generalized to the context of T-free relational structures in the sense of Herwig-Lascar. In particular, we show that some results of Coulbois and Rosendal can be generalized to groups with the Herwig-Lascar property. Finally, we show that groups with the Herwig-Lascar property are closed under taking free products.

Jun Le Goh: *A Σ^1_1 axiom of finite choice and Steel forcing*

Abstract: Several variants of the $Sigma^1_1$ axiom of choice are known to be theories of hyperarithmetic analysis, i.e., all of their ω-models are closed under hyperarithmetic reduction and they hold in $\mathrm{HYP}(Y)$ for every $Y \subseteq \omega$. Most theories of hyperarithmetic analysis are linearly ordered by provability, with the only known exception being the arithmetic Bolzano-Weierstrass theorem (Conidis). We introduce a Σ^1_1 axiom of finite choice and use a variant of Steel forcing to show that it also lies to the side, strengthening results of Conidis.

Valentina Harizanov: *Orderable magmas*

Abstract: We investigate algebraic and computability-theoretic properties of orderable magmas. A magma is an algebraic structure with a single binary operation. Important examples of magmas include semigroups and groups, but also those that come from knot theory and do not necessarily have an associative operation, such as racks and quandles. A magma M is right-orderable (left-orderable) if there is a linear ordering of the domain of M, which is right-invariant (left-invariant) with respect to the magma operation. If M has a left order that is also a right order, then M is bi-orderable. A computable orderable magma does not necessarily have a computable order. There is a natural topology on the set of all (left, right, bi-) orders of M. These spaces are compact, and in some cases homeomorphic to the Cantor set. This is joint work with J. Chubb and T. Ha.

Denis Hirschfeldt: *Reduction games, provability, and compactness*

Abstract: Notions of computability-theoretic reducibility between mathematical principles can be used to give an analysis of the relative strength of such principles complementary to the proof-theoretic perspective of reverse mathematics. Computable reducibility and Weihrauch reducibility are two such notions. Hirschfeldt and Jockusch introduced reduction games as a way to define generalizations of these reducibilities. In their original forms, these games are played over the standard natural numbers, but Hirschfeldt and Jockusch also suggested the possibility of extending them to nonstandard models of fragments of **PA**. In particular, they showed that doing so for generalized computable reducibility results in a characterization of implication over $\mathbf{RCA}_0$, the usual weak base system of reverse mathematics. I will discuss joint work with Damir D. Dzhafarov and Sarah C. Reitzes in which we build on this idea in several directions.

Neil Lutz: *New applications of point-to-set principles*

Abstract: This talk will describe new ways that algorithmic fractal dimensions have been used to answer open questions in classical fractal geometry. Point-to-set principles empower these dimension concepts, which are constructs of the theory of computing and can be characterized using Kolmogorov complexity, to address problems of mathematical analysis whose statements do not involve the theory of computing.

Maciej Malicki: *Complexity of the isomorphism relation for metric structures*

Abstract: For a given sentence s in the infinitary logic, one can consider the Borel space M of all countable models satisfying s, and the isomorphism relation E on M. G. Hjorth and A. Kechris found a model-theoretic characterization of the situations when E is smooth or essentially countable. We generalize their results to the context of Polish metric structures and continuous infinitary logic. We also give a short proof of a theorem of A. Kechris to the effect that orbit equivalence relations induced by continuous actions of locally compact groups are essentially countable. This is joint work in progress with A. Hallback and T. Tsankov.

Joe Miller: *Highness properties close to PA-completeness*

Abstract: The PA degrees have played an interesting supporting role in the study of algorithmic randomness, in part because they allow us to approximate certain objects that play a central role. Examples include plain Kolmogorov complexity, prefix-free complexity, and the optimal supermartingale. All three are intrinsically c.e. objects that are optimal in their classes, they are all Turing equivalent to $\emptyset$, and they can all be approximated using PA degrees. Some such approximation problems turn out to require a PA degree, and some do not. Among the problems we separate from PA-completeness, the strongest is computing a subtree of positive measure with no dead ends of a given Π^0_1 tree of positive measure. We call this the *continuous covering property*. We also separate the corresponding principle from WKL in reverse mathematics.

Russel Miller: *Uniform computable categoricity and Scott families*

Abstract: The original concept of computable categoricity applied only to computable structures. The subsequent notion of relative computable categoricity was intended to generalize this concept to noncomputable structures, and generally succeeded in doing so. Additionally, relative computable categoricity yielded the equivalent syntactic concept of having a computably enumerable Scott family of Σ_1 formulas over a finite tuple of constants, as shown by Ash, Knight, Manasse, and Slaman (and independently by Chisholm). Uniform computable categoricity, as studied by Ventsov and later by Downey, Hirschfeldt, and Khoussainov, is equivalent to the same syntactic condition with no constants allowed. However, all of the results above assume that the structure in question is computably presentable. For the relative and uniform notions, this restriction does not seem appropriate, as these concepts specifically include all copies of the structure (on the domain ω), computable and noncomputable. We will generalize them to apply equally well to structures with no computable presentation, giving examples to illustrate all these ideas, mentioning situations where the new notions have found uses, and adapting the theorem on Scott families to this new context.

Andrey S. Morozov: *On Σ–preorderings in $HF(\mathbb{R})$*

Abstract: We prove that ω_1 cannot be embedded into any preordering Σ–definable with parameters in the hereditarily finite superstructure over the ordered field of real numbers, $HF(\mathbb{R})$. As corollaries, we obtain characterizations of Σ–presentable ordinals and Gödel constructive sets of kind L_α. It also follows that there are no Σ–presentations for structures of T–, m–, 1–, and tt–degrees over HF(R).

Aristotelis Panagiotopoulos: *Definable (co)homology and classification of solenoids*

Abstract: We will develop a framework for enriching various classical invariants from algebraic topology with descriptive set-theoretic information. Applying these ideas to Steenrod homology theory we get a new invariant for compact metrizable spaces up to homotopy equivalence which we call "definable homology." Similarly we get a dual notion of "definable cohomology" for locally compact metrizable spaces by enriching the classical Čech cohomology theory. Our invariants are strictly finer than the original ones. In particular, we will show that n-dimensional (co)solenoids are completely classified up to homeomorphism by their definable (co)homology. In the process, we will generalize Veličković's rigidity theorem for definable automorphisms of $\mathcal{P}(\omega)/\mathrm{fin}$, to the arbitrary quotient G/N, where G is a locally pro-finite abelian group and N is a countable dense subgroup of G, and we will develop some basic definable homological algebra.

Christopher Porter: *Intersecting algorithmically random closed sets*

Abstract: The intersections of pairs of algorithmically random closed subsets of Cantor space was initially studied by Cenzer and Weber. In this talk, I will discuss recent work with Adam Case in which we extend the analysis of Cenzer and Weber, answering an open question they posed and extending their analysis to multiple intersections of random closed sets.

Jan Reimann: *Measures, Randomness, and Domination*

Abstract: We introduce a family of Hausdorff measures based on the dissipation function of a continuous probability measure, parameterized by a natural number n. The corresponding Solovay tests then induce an ω-hierarchy of randomness tests with respect to a continuous measure. We show that there exists a close connection between this test hierarchy and the ability of a real to admit a modulus function. We use this connection to prove the existence of NCR elements in various families of Turing degrees. This is joint work with Mingyang Li.

Dino Rossegger: *The complexity of embeddings between linear orders and Boolean algebras*

Abstract: We investigate the computational complexity of embeddings between bi-embeddable linear orders and Boolean algebras. This investigation is particularly interesting for these classes of structures as they have the property "hyperarithmetic is recursive", i.e., every hyperarithmetic Boolean algebra or linear order is bi-embeddable with a computable one. Our main results show that the computational complexity of embeddings between computable bi-embeddable linear orders of finite Hausdorff rank is determined by their Hausdorff rank. We furthermore obtain results on the complexity of embeddings between computable bi-embeddable Boolean algebras. This is joint work with Nikolay Bazhenov and Maxim Zubkov.

Forte Shinko: *Quotients by countable subgroups are hyperfinite*

Abstract: Given a countable group Γ, the outer automorphism group $Out(\Gamma)$ is either countable or of cardinality continuum. A finer and more suitable notion is to consider the Borel complexity of $Out(\Gamma)$ as a Borel equivalence relation. We show that in this context, $Out(\Gamma)$ is of rather low complexity, namely that it is a hyperfinite Borel equivalence relation. In general, we show that for any Polish group G and any countable normal subgroup Γ, the quotient group G/Γ is hyperfinite. This is joint work with Joshua Frisch.

Levi Sledd: *Assouad-Nagata dimension of $C'(1/6)$ groups*

Abstract: Asymptotic dimension is a coarse invariant of metric spaces, introduced by Gromov in 1993 as a large-scale analogue of topological dimension. A related concept is that of Assouad-Nagata dimension, a quasi-isometry invariant which is bounded below by asymptotic dimension. Historically, these two invariants have been hard to distinguish among finitely generated groups. In this talk, we show that any finitely generated $C'(1/6)$ group has Assouad-Nagata dimension at most 2. Using this result we show how to construct, for any $n, k \in \mathbb{N}$ with $n \geq 3$, a finitely generated group of asymptotic dimension n and Assouad-Nagata dimension $n + k$.

Slawomir Solecki: *Dynamics of Polish groups, concentration of measure, and submeasures*

Abstract: Topological dynamics of Polish groups has interesting aspects not present in dynamics of locally compact groups. For example, there exist Polish groups whose all continuous actions on compact spaces have fixed points; in fact, the unitary group of the separable, infinite dimensional Hilbert space is such. Groups of this type, called extremely amenable, were first constructed by Herer and Christensen using certain submeasures. Later, Gromov and Milman made a connection between extreme amenability and the concentration of measure phenomenon from probability theory.

I will describe these developments. In this context, I will present a new concentration of measure theorem. I will describe the dynamical consequences, in the spirit of Gromov and Milman, of our concentration of measure theorem. These consequences generalize the Herer–Christensen result mentioned above as well as related results of

Glasner and Pestov. All this depends on a new “geometric” classification of submeasures, which I will outline. This is a joint work with F. Martin Schneider.

Mariya Soskova: *Fragments of the theory of the enumeration degrees*

Abstract: I will present recent joint work with Ted Slaman and Steffen Lempp. We consider the problem of finding the quantifier level where the theory of the partial order of the enumeration degrees becomes undecidable. It is well known that the existential theory is decidable. We establish that the 3-quantifier theory is undecidable. We show that a fragment of the 2-quantifier theory, known as the extension of embeddings problem is decidable and discuss possible approaches and obstacles towards a decision procedure for the full 2-quantifier theory.

Donald Stull: *Effective dimension and projections*

Abstract: Effective dimensions, introduced by Jack Lutz in the early 2000s, use computability theory to quantify the fine-grained randomness of individual points in $\mathbb{R}^n$. Recent results have shown that algorithmic information techniques can answer questions in geometric measure theory. In this talk, we discuss recent and ongoing work on this topic. In particular, we will focus on the application of effective dimnension to *projection problems*. Determining the (classical) fractal dimensions of a set projected onto a subspace is a central theme of geometric measure theory. We will show that algorithmic techniques can shed light on these questions, as well as discuss directions for future research.

Henry Towsner: *Why is there only one way for a matrix to be random?*

Abstract: We can think of the eigenvalues of a matrix as characterizing how “structured” it is: a matrix is “more structured” if it has a few large eigenvalues, and “close to random” if all but one of its eigenvalues are small. With tensors, the picture becomes more complicated: we can identify several different kinds of “eigenvalues” and “eigenvectors”, and identify tensors which are “random” for one kind of eigenvector but “non-random” for a different kind. Each notion of eigenvector captures one kind of “structure” a tensor might have.

Joseph Zielinski: *Isomorphism of compact structures*

Abstract: An argument of A.S. Kechris and S. Solecki shows that the homeomorphism relation between compact Polish spaces is Borel reducible to the orbit equivalence relation of a Polish group action. By adapting this argument, we see the same is also true for the homeomorphic isomorphism relation between compact structures—that is, of compact Polish spaces, each equipped a countably family of closed relations.

From this, we bound the complexity of some other types of (not necessarily compact) structures by associating, to each, an appropriate compact structure. In every case where this is accomplished, this complexity is at or below that of the orbit equivalence relation of some Polish group action. For certain structures, this process delivers stronger bounds, by limiting the topological features of the associated compact structures, and thereby simplifying the complexity of the corresponding orbit equivalence relation. This is joint work with C. Rosendal.

Jenna Zomback: *A combinatorial proof of the pointwise ergodic theorem for actions of amenable groups along Tempelman Følner sequences*

Abstract: A pointwise ergodic theorem for the action of a countable group Γ on a probability space equates the global ergodicity of the action to its pointwise combinatorics. Our main result is a short, combinatorial proof of the pointwise ergodic theorem for actions of amenable groups along Tempelman Følner sequences, which is a slightly less general version of Lindenstrauss’s celebrated theorem. In fact, we prove that such actions have a certain tiling property, which implies the pointwise ergodic theorem. This is joint work with Jon Boretsky.