SEALS 2021 BOOK OF ABSTRACTS

Speaker: Chris Conidis

Title: An algebraic equivalent to a combinatorics of minimal primes in Noetherian rings

Abstract: We will briefly introduce a combinatorial theorem, namely the Tree-Antichain Theorem (TAC), that falls outside of the Big Five subsystems of Second-Order Arithmetic. Then we will produce an algebraic statement regarding minimal prime ideals in Noetherian rings that characterizes TAC over RCA_0.

Speaker: Natasha Dobrinen

Title: Ramsey theory on infinite structures

Abstract: The Infinite Ramsey Theorem says that for any positive integer n, given a coloring of all n-element subsets of the natural numbers into finitely many colors, there is an infinite set M of natural numbers such that all n-element subsets of M have the same color. Infinite Structural Ramsey Theory is concerned with finding analogues of the Infinite Ramsey Theorem for Fraisse limits, and also more generally for universal structures. In most cases, the exact analogue of Ramsey's Theorem fails. However, sometimes one can find bounds of the following sort: Given a finite substructure A of an infinite structure S, we let T(A,S) denote the least number, if it exists, such that for any coloring of the copies of A in S into finitely many colors, there is a substructure S' of S, isomorphic to S, such that the copies of A in S' take no more than T(A,S) colors. If for each finite substructure A of S, this number T(A,S) exists, then we say that S has "finite big Ramsey degrees".

In the past six years, there has been a resurgence of investigations into the existence and characterization of big Ramsey degrees for infinite structures, leading to many new and exciting results and methods. We will present an overview of the area and some highlights of recent work by various author combinations from among Balko, Barbosa, Chodounsky, Coulson, Dobrinen, Hubicka, Konecny, Masulovic, Patel, Vena, and Zucker.

Speaker: Damir Dzafarov

Title: Milliken's tree theorem and computability theory

Abstract: Milliken's tree theorem is a powerful combinatorial result that generalizes Ramsey's theorem and many other familiar partition results. I will present recent work on the effective and proof-theoretic strength of this theorem, which was originally motivated by a question of Dobrinen. The main result is a complete characterization of Milliken's tree theorem in terms of reverse mathematics and the usual computability-theoretic hierarchies, along with several applications to other combinatorial problems.

Key to this is a new inductive proof of Milliken's tree theorem, employing an effective version of the Halpern-Lauchli theorem. This is joint work with Angles d'Auriac, Cholak, Monin, and Patey.

Speaker: Marta Fiori Carones

Title: The strength of ADS and CAC in the absence of Σ^0_1-induction

Abstract: This is joint work with Leszek Kołodziejczyk and Katarzyna Kowalik. The common base theory of reverse mathematics is the theory RCA_0, which guarantees the existence of Δ^0_1-definable sets and where induction for Σ^0_1-formulae holds. Simpson and Smith in 1986 introduced a different base theory, RCA_0*, where induction is weakened to Δ^0_1-formulae. In more recent years Leszek Kołodziejczyk, Katarzyna Kowalik, Tin Lok Wong and Keita Yokoyama started wondering about the strength of Ramsey's theorem over RCA_0*. In this talk further results in this direction are presented. In particular we concentrate on the Ascending Descending Sequence principle and on the Chain/Antichain principle, well known consequences of Ramsey's theorem for pairs. Various formalisations of these principles are available in the literature and proved to be equivalent over RCA_0. This is not always the case over RCA_0*, as some formulations can be separated over the weaker base theory, though the overall picture is still quite neat.

Speaker: Joshua Frisch

Title: Embedding theorems for Polish Modules
Abstract: A Polish module is a topological module whose underlying topology is Polish. In this talk I will discuss some recent work (joint with Forte Shinko) where we study when uncountable Polish modules continuously inject into one another and the pre-order induced by these injections. In particular we show that, for a wide class of rings, there are countably many minimal elements in this pre-order. As an application, we will construct a countable family of uncountable abelian Polish groups, at least one of which embeds into any other uncountable abelian Polish group.

Speaker: Jeff Hirst

Title: Weihrauch analysis motivated by finite complexity theory (Preliminary results)

Abstract: We apply formalized Weihrauch analysis to some combinatorial principles related to formulations of LPO and traditional Weihrauch analysis to some much stronger principles. Asuka Wallace and Zachery Bement contributed to this work.

Speaker: Jan Hubička

Title: Higher order dualization of the Ramsey theorem and big Ramsey degrees

Abstract: Extending talk of Matěj Konečný we discuss a strengthening of the infinitary dual Ramsey theorem (Carlson-Simpson theorem) for words indexed by multi-dimensional sets we call "higher order

words". This theorem can be applied to give upper bounds on Big Ramsey degrees of free amalgamation classes in finite binary language with finitely many irreducible constraints. This presents an alternative approach to recent results of Andy Zucker and Natasha Dobrinen. We will also discuss generalization to some strong amalgamation classes such as those in Cherlin's catalogue of metrically homogeneous graphs. This is a joint work with Martin Balko, Natasha Dobrinen, David Chodounský, Matěj Konečný, Jaroslav Nešetřil, Stevo Todorcevic, Lluis Vena and Andy Zucker.

Speaker: Meng-Che Turbo Ho

Title: 0-1 laws for finitely presented structures

Abstract: Random groups are proposed by Gromov as a model to study the typical behavior of finitely presented groups. They share many properties of the free group, and Knight conjectured that random groups satisfy a strong zero-one law and have the same first-order theory as the free group. In joint work with Franklin and Knight, we study this zero-one law in other classes of structures. In particular, we consider random presentations in algebraic varieties in the sense of universal algebra. We will discuss some examples where the zero-one law holds and some other examples where the zero-one law fails. We will also discuss some general results.

Speaker: Matěj Konečný

Title: Big Ramsey degrees for metric spaces with finitely many distances

Abstract: We give a short proof that every countable (ultra)homogeneous metric space which uses only finitely many distances has finite big Ramsey degrees. This is an adaptation of Hubička's recent proof of finite big Ramsey degrees for the triangle-free graph using the Carlson-Simpson theorem. We also illustrate what is needed to extend this method to structures with higher-arity relations. This is part of an ongoing project with Aranda, Balko, Dobrinen, Hubička, Chodounský, Nešetřil, Vena, and Zucker.

Speaker: Alex Kruckman

Title: Higher dimensional obstructions for star reductions

Abstract: The Becker graph is a directed graph structure on the set of orbits of the action of a Polish group G on a Polish space X. In the case of the logic action of the infinite symmetric group on the space of countable L-structures, the Becker graph is exactly the embeddability relation between isomorphism classes of L-structures. Building on work of Lupini and Panagiotopoulos, we show that a reduction between orbit equivalence relations which is Baire measurable and category-preserving (we call such a reduction a "star reduction") induces generically an embedding between their Becker graphs. This allows us to obstruct star reductions using invariants associated to the Becker graph. As an application of one such invariant, a notion of dimension related to higher amalgamation properties, we exhibit an infinite

family of orbit equivalence relations which are pairwise incomparable with respect to star reductions. This is all joint work with Aristotelis Panagiotopoulos.

Speaker: Jamal Kawach

Title: Fraïssé and Ramsey properties of Fréchet spaces

Abstract: In this talk we will survey some recent results concerning Fraïssé Banach spaces and their associated groups of surjective linear isometries. We will then see how to extend the relevant Fraïssé theory to the setting of topological vector spaces equipped with a sequence of seminorms, and in particular to Fréchet spaces. We conclude with a discussion of the approximate Ramsey property and its relation to the topological dynamics of the automorphism groups of Fraïssé Fréchet spaces. This talk contains joint work with Jordi López-Abad.

Speaker: Christopher Lambie-Hanson

Title: Nontrivial coherent families of functions

Abstract: In the 1980s, considerations in homological algebra gave rise to combinatorial set theoretic questions about nontrivial coherent families of functions indexed by elements of the Baire space. The existence of such families turned out to be intimately connected to cardinal characteristics of the continuum and the Open Coloring Axiom. Similar homological considerations naturally give rise to higher-dimensional analogues of these nontrivial coherent families of functions. We will begin the talk by introducing these families and indicating how they arise from algebraic questions about the derived functors of the inverse limit functor. We will then sketch a proof of the fact that, in the forcing extension obtained by adding $\beth_\omega$-many Cohen reals, for every $n>0$, every n-dimensional coherent family of functions indexed by $(\omega^\omega)^n$ is trivial. We end with a broader discussion of some of the implications of this result and its techniques and of directions for further research. This is joint work with Jeffrey Bergfalk and Michael Hrušák.

Speaker: Patrick Lutz

Title: Two Applications of a New Basis Theorem for Perfect Sets

Abstract: In recent work with Benny Siskind, we proved that part 1 of Martin's conjecture holds for order preserving functions on the Turing degrees. A key component of the proof is a new basis theorem for perfect sets which strengthens an older theorem of Groszek and Slaman. I will explain this basis theorem along with two applications. First, the application to Martin's conjecture mentioned above. Second, a curious phenomenon about embedding partial orders into the Turing degrees: every locally countable Borel partial order of height two has a Borel embedding into the Turing degrees, but the same is not true for height three. This second application is joint work with Kojiro Higuchi.

Speaker: Andrew Marks

Title: Increasing unions of countable Borel equivalence relations, and a question of Slaman and Steel

Abstract: An important way to study the complexity of Borel equivalence relations is in terms of whether they can be written as increasing unions of simpler equivalence relations. For example, the equivalence relations that can be written as increasing unions of Borel equivalence relations with finite classes are called the hyperfinite Borel equivalence relations. They are an important and well-studied class. We consider an old question of Slaman and Steel: whether Turing equivalence is an increasing union of Borel equivalence relations none of which contain a uniformly computable infinite sequence. We show this question is deeply connected to problems surrounding Martin's conjecture, and also in countable Borel equivalence relations. This is joint work with Adam Day.

Speaker: Antonio Montalbán

Title: Transfinite Ramsey Theorem.

Abstract: We consider a version of the Ramsey Theorem for coloring tuples within a finite set where the exponent is transfinite. That is, the tuples we color are γ-large for some ordinal γ. As in Ramsey theorem we ask: How large should a finite set of numbers be to ensure that, for all colorings of the γ-large tuples with a certain finite number of colors, there exists a homogeneous set that is α-large? Again, α being an ordinal. The answer should be an ordinal, and its existence follows from the Galvin-Prikry Theorem. What we ask is about the precise value of the bound, given as a function of the ordinals α and γ. We also consider computability theoretic and reverse mathematics issues related to this.

This is joint work with Alberto Marcone

Speaker: Carl Mummert

Title: König's edge coloring theorem and related results

Abstract: I will discuss the Reverse Mathematics of König's edge coloring theorem, which says a bipartite graph with maximal degree n has an edge coloring with no more than *n* colors. Interestingly, the number of colors permitted affects the computability of the solution. Computable bipartite graphs with degree bounded by *n* have computable edge colorings with *2n-1* colors, but the theorem that there is an edge coloring with *n* colors is equivalent to WKL_0 over RCA_0. I will also discuss closely related work of Cenzer, Schmerl, Hirst, Shafer, and others, and a few interesting open questions in computable graph theory.

Speaker: Miriam Parnes

Title: A Generalization of Tree Decomposition to Amalgamation Classes of Finite Structures

Abstract: The tree width of a graph is found from its tree decompositions and is an important measure of how connected the graph is. Many questions which are hard to answer about graphs in general become easy to answer about classes of graphs when their tree width is bounded. It was shown by Seymour and Thomas in 1993 that for a particular game of Cops and Robbers played on graphs, *k* cops have a winning strategy for the game on the graph *G* if and only if *G* has tree width less than *k*. Since the class of all finite graphs is an example of a type of amalgamation class called a Fraissé class, we would like to extend the notions of tree decomposition, tree width, and the Cops and Robbers game to other Fraissé classes. In this talk, I will explain how we can do this for Fraissé classes with a particular kind of independence relation.

Speaker: Rehana Patel

Title: The number of ergodic models of an infinitary sentence

Abstract: Given an $L_{\omega 1\omega}$-sentence ϕ in a countable language, we call an ergodic S_∞-invariant probability measure on the Borel space of countable models of ϕ (having fixed underlying set) an *ergodic model* of ϕ. I will discuss the number of ergodic models of such a sentence ϕ, including the case when ϕ is a Scott sentence. This is joint work with N. Ackerman, C. Freer, A. Kruckman and A. Kwiatkowska.

Speaker: Ludovic Patey

Title: Classifications of Ramsey-like theorems

Abstract: Ramsey's theorem asserts that every k-coloring of $[\omega]^n$ admits an infinite monochromatic set. There exists a 2-coloring of $[\omega]^2$ whose solutions compute the halting set. On the other hand, this property becomes false if one weakens the homogeneity requirement to transitivity (Erdos-Moser theorem) or to more colors (Thin set theorem).For these weaker statements, the property fails in a strong way, known as strong cone avoidance. In this talk, introduce a natural class of Ramsey-like theorems encompassing many variants of Ramsey's theorem studied in reverse mathematics. We show that this class admits a maximal statement satisfying strong cone avoidance and use it as a criterion to re-obtain many existing proofs of (strong) cone avoidance. We also discuss a new program of classification of Ramsey-like theorems based on various computability-theoretic preservation properties.

Speaker: Arno Pauly

Title: The non-computability of finding Nash equilibria in multiplayer games

Abstract: In game theory, the notion of a Nash equilibrium is put forth as the quintessence of rational behaviour. Alas, if we allow payoffs to be real numbers, computing a Nash equilibrium of a given finite game in normal form is not possible in general. To gauge how big of a problem this is, we want to understand the degree of non-computability. The suitable degree structure here are the Weihrauch degrees. For one and two player games, a precise classification was already established in previous work. Here, we explore what happens in multiplayer games. Along the way, we explore the degree of finding roots of polynomials. This is joint work with Tonicha Crook.

Speaker: Sarah Reitzes

Title: Reduction games over RCA_0

Abstract: In this talk, I will discuss joint work with Damir D. Dzhafarov and Denis R. Hirschfeldt. Our work centers on the characterization of problems P and Q such that $P \leq_\omega Q$, as well as problems P and Q such that RCA_0 proves $Q \rightarrow P$, in terms of winning strategies in certain games. These characterizations were originally introduced by Hirschfeldt and Jockusch. I will discuss extensions and generalizations of these characterizations, including a certain notion of compactness that allows us, for strategies satisfying particular conditions, to bound the number of moves it takes to win. This bound is independent of the instance of the problem P being considered. This allows us to develop the idea of Weihrauch and generalized Weihrauch reduction over some base theory. Here, we will focus on the base theory RCA_0. In this talk, I will explore these notions of reduction among various principles, including bounding and induction principles.

Speaker: Diego Rojas

Title: Toward an effective theory of weak convergence of measures

Abstract: In the available computable measure theory literature, there exist examples of uniformly computable sequences of measures that converge weakly to an incomputable measure. This suggests that an effective theory of weak convergence of measures needs further study. To this end, we propose two effective notions of weak convergence of measures on the real line: one uniform and one non-uniform. We show that these notions are equivalent for uniformly computable sequences of measures. We then discuss an effective version of a Portmanteau Theorem, a characterization theorem for weak convergence of measures.

Speaker: Lynn Scow

Title: Generalized indiscernible sequences and tame *k*-colorings

Abstract: We explore certain Fraissé classes that have a partition property for a class of tame colorings. Arguments will use tools from model theory.

Speaker: Reed Solomon

Title: Dominating orders, vertex pursuit games and effectiveness

Abstract: Graph theorists have studied many variants of the vertex pursuit game in which two players take turns moving on a graph and Player I wins if she ever occupies the same vertex as Player II. For finite graphs, Player I has a winning strategy if and only if the graph admits a dominating order. This connection breaks down for infinite graphs. Graph theorists have proposed several variants which recover some aspects of this connection. In this talk, I will give some computability theoretic results about dominating orders and use similar techniques to give a negative answer to an open question about one of these variants.

Speaker: Spencer Unger

Title: Borel embeddings and factor maps between actions of Z^d

Abstract: Motivated by results in ergodic theory, we present several results on the existence of Borel embeddings and factor maps from actions of Z^d into natural spaces of colorings, tilings and Hamilton paths of the Cayley graph of Z^d.This is joint work with Nishant Chandgotia.

Speaker: James Walsh

Title: Reducing omega-model reflection to iterated syntactic reflection

Abstract: Two types of principles are commonly called “reflection principles” in reverse mathematics. According to syntactic reflection principles for T, every theorem of T (from some complexity class) is true. According to semantic reflection principles, every set belongs to some (sufficiently correct) model of T. We will present a connection between syntactic reflection and semantic reflection in second-order arithmetic: For any Π^1_2 axiomatized theory T, every set is contained in an omega-model of T if and only if every iteration of Π^1_1 reflection for T along a well-ordering is Π^1_1 sound. There is a thorough proof-theoretic understanding of the latter in terms of ordinal analysis. Accordingly, this reduction yields proof-theoretic analyses of omega-model reflection principles. This is joint work with Fedor Pakhomov.

Speaker: Linda Brown Westrick

Title: Lowering dimension with few changes

Abstract: Consider Cantor space with the Besicovitch pseudo-metric, defined as the (limsup of) the density of the places where *X* and *Y* differ. Because effective dimension varies continuously with respect to this pseudo-metric, we can try to get a clearer picture of how they relate. Given a set *Y* of effective dimension *s*, and some *t* distinct from *s*, how far away is the nearest *X* of effective dimension *t*? If the answer depends on *Y* (it does), what are the best case and the worst case situations? The case where $t < s$ was left open by Greenberg, Miller, Shen and the author in 2018. We find optimal bounds for the $t<s$ case and describe the kinds of Y which attain these bounds. Joint with Goh, Miller & Soskova.