ANALYTIC EQUIVALENCE RELATIONS WITH ALL BOREL CLASSES ARE BOREL SOMEWHERE

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Abstract. The question of interest in this talk is whether certain equivalence relations on Polish spaces can be a $\Delta^1_1$ equivalence relation when restricted to certain subsets.

As every equivalence relation is $\Delta^1_1$ when restricted to countable subsets, this triviality can be removed by requiring the subsets to be non-small according to $\sigma$-ideals. This question will be considered for the large class of $\sigma$-ideals whose forcing of positive $\Delta^1_1$ subsets is a proper forcing. A positive answer can only be feasible if the equivalence relations bear at least some resemblance to $\Delta^1_1$ equivalence relations. Hence, Kanovei, Sabok, and Zapletal asked the following precise question: Is every $\Sigma^1_1$ equivalence relation with all $\Delta^1_1$ classes a $\Delta^1_1$ equivalence relation on some $\Delta^1_1 I^+$ set, whenever $I$ is a $\sigma$-ideal whose associated forcing is a proper forcing.

This talk will discuss this question and how to use a measurable cardinal (or sharps of certain sets) to give a positive answer.

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