Exam 2.

Due Wednesday Nov. 1 before class.

1. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a strictly increasing function ($r < s$ implies $f(r) < f(s)$). Show that the set $\{r \in \mathbb{R}\colon f \text{ is not continuous at } r\}$ is countable. *Hint.* Produce an injection of the set into rationals.

2. Produce a function $f\colon \mathbb{R} \to \mathbb{R}$ such that the set $\{r \in \mathbb{R}\colon f \text{ is not continuous at } r\}$ is uncountable.

3. Let $\langle P, \leq \rangle$ be a partially ordered set. Define a transfinite recursive process by $P_0 = P$, $P_{\alpha+1} = P_\alpha \setminus \{p \in P_\alpha\colon p \text{ is minimal in } P_\alpha\}$, and $P_\alpha = \bigcap_{\beta\in\alpha} P_\beta$ for a limit ordinal α. Show that there is an ordinal α such that $P_\alpha = P_{\alpha+1}$.

4. Show that the set $P_\alpha = P_{\alpha+1}$ from (3) is the inclusion-largest subset of P which has no minimal elements.

5. Prove that every filter on ω can be extended to an ultrafilter.