Exam 2.

Due Wednesday Nov. 22 if sent by e-mail, or Monday Nov. 20 in class if handed in on paper.

1. Let $\langle A, < \rangle$ and $\langle B, \prec \rangle$ be two countable dense linear orders without endpoints. Let $f\colon A \to B$ be a function with finite domain, preserving the ordering. Show that there is an isomorphism $g\colon A \to B$ which extends f (in other words, $f \subset g$, or $\forall a \in \operatorname{dom}(f)\ f(a) = g(a)$).

2. Show that if F is an Archimedean field, then the cardinality of F is less or equal to the cardinality of the powerset of ω. *Hint.* Show that an element F is uniquely determined by the set of all F-rationals smaller than it.

3. A set $A \subset \mathbb{R}$ is *analytic* if it is a projection of some Borel set $B \subset \mathbb{R} \times \mathbb{R}$ into the first coordinate. Show that the set $\{A \subset \mathbb{R} \colon A \text{ is analytic}\}$ has the same cardinality as $\mathbb{R}$. Conclude that there are subsets of $\mathbb{R}$ which are not analytic.

4. Let B_n for $n \in \omega$ be dense G_δ sets of reals. Use the Baire category theorem to show that $\bigcap_n B_n \neq 0$.