

Exam 1.

The solutions are due Wednesday February 7th before class. Either bring them to class, or send them by e-mail to my address (preferably in pdf).

1. Recall that if E is an equivalence relation on a set X , then a *selector* for E is a set $S \subset X$ which contains exactly one element from every E -equivalence class. Show that the Axiom of Choice is equivalent to the statement that every equivalence relation has a selector.
2. Let x be a set, and by transfinite recursion define $x_0 = x$, $x_{\alpha+1} = x_\alpha \setminus \{y : y \cap x_\alpha = 0\}$, and $x_\alpha = \bigcap_{\beta \in \alpha} x_\beta$ if α is limit. Show that there is an ordinal α such that $x_\alpha = 0$.
3. Is there a cardinal κ such that the cofinality of κ is ω_1 ? Justify your answer.
4. Let κ be an uncountable regular cardinal. show that the set of limit ordinals in κ is a closed unbounded subset of κ .