Exam 1.

The solutions are due Wednesday Oct. 17 before class. Either leave them under my office door (456 Little Hall) or with the receptionist at 358 Little Hall, or send them by e-mail to my address (preferably in pdf).

For all problems, let X be an arbitrary Polish space with a compatible complete metric d. We will go through the proof showing that the space Z of all isometries of X with the pointwise convergence topology is a Polish space. Let $A \subset X$ be a countable dense set and let Y be the space of all functions from A to X with the product topology.

1. Prove that the set $\{y \in Y : \mathrm{rng}(y) \subset X \text{ is dense}\}$ is G_δ in Y.

2. Prove that the set $\{y \in Y : y \text{ preserves the metric } d\}$ is closed in Y.

3. Show that every function $y \in Y$ which preserves the metric and such that $\mathrm{rng}(y)$ is dense can be extended to a unique isometry of the space X. Write $Y_0 = \{y \in Y : y \text{ preserves the metric and } \mathrm{rng}(y) \text{ is dense}\}$. Conclude that the map $\pi \colon Z \to Y_0$ given by $\pi(z) = z \restriction A$ is a bijection.

4. Let $z, z_n : n \in \omega$ be elements of Z. Show that z_n converge pointwise to z just in case $z_n \restriction A$ converge to $z \restriction A$ in Y. Conclude that the map $\pi \colon Z \to Y_0$ from Problem 3 is a homeomorphism between Z with the pointwise convergence topology and Y_0 with the topology inherited from Y. Conclude that the space Z is Polish.