Exam 1.

The solutions are due Wednesday February 13th before class. Either bring them to class, or send them by e-mail to my address (preferably in pdf).

1. (Projective space.) Let $n > 0$ be a natural number, let $X = \mathbb{R}^n$ and let E be the equivalence relation connecting x_0, x_1 if there is a nonzero scalar $r \in \mathbb{R}$ such that $x_0 = rx_1$. Is E smooth? Does it have a transversal?

2. (Affine space.) Let $n > 0$ be a natural number, let $X = (\mathbb{R}^n)^2$ and let E be the equivalence relation on X connecting $\langle a, b \rangle$ to $\langle c, d \rangle$ if $a - c = b - d$ (coordinatewise operations). Is E smooth? Does it have a transversal?

3. Let G be a compact Polish group, acting continuously on a Polish space X. Show that the induced orbit equivalence relation is smooth. *Hint.* For every $x \in X$, the orbit of x is compact, and therefore an element of the Polish hyperspace $K(X)$.