Exam 1.

No cooperation. The solutions are due Monday February 10 before class.

1. Prove without the use of the completeness theorem that in propositional logic, $\vdash \phi \to \neg\neg\phi$ and $\vdash \neg\neg\phi \to \phi$ are both true.

2. Consider the theory of dense linear order without endpoints. Enrich its language by two constant symbols c, d, and add to it the sentence $c < d$. Prove that the resulting theory is complete.

In the following problems, a graph is a pair $\langle V, E\rangle$ where V is a set (of vertices) and $E \subset [V]^2$ is a set (of edges). Graphs are models for a first order language $\mathfrak{L}$ with a single binary relational symbol interpreted as xRy if x, y are connected with an edge. A cycle of length n is a sequence $x_0, x_1, \dots x_{n-1}$ of pairwise distinct vertices such that x_i, x_{i+1} are connected with an edge for all $i < n-1$ and also the vertices x_0, x_{n-1} are connected with an edge. A graph is then a model of the theory Γ consisting of the sentences $\forall x, y\ x\ R\ y \leftrightarrow y\ R\ x$ and $\forall x\ \neg x\ R\ x$.

3. For every natural number n, write a first order sentence in $\mathfrak{L}$ equivalent to "there are no cycles of length n".

4. Show that there is no first order sentence in $\mathfrak{L}$ equivalent to "there are no cycles".

5. Find some sentence ϕ such that the theory Γ, ϕ is complete (decides every sentence).