Exam 1.

The solutions are due Wednesday March 14th before class. Either bring them to class, or send them by e-mail to my address (preferably in pdf).

1. Consider the structure $\langle \mathbb{Q}, +, \leq \rangle$ of rationals with the usual addition and ordering. Consider the sets $A_0 = \{0\}$, A_1 =positive rationals, A_2 =negative rationals. Show that A_0, A_1, A_2 and their boolean combinations are the only sets definable without parameters in the structure.

2. Find an example of a linear order $\langle L, \leq \rangle$ and a set $A \subset L$ such that A is definable without parameters in $\langle L, \leq \rangle$ and it is not a finite union of open intervals and singletons.

3. Consider the symmetric binary relation R on integers defined by $x \ R \ y$ if $|x - y| = 1$. Describe an algorithm deciding whether a given sentence holds in the structure $\langle \mathbb{Z}, R \rangle$. *Hint.* Define the relation from the ordering of integers.

4. In the structure of problem 3, can one define the usual ordering of integers in the structure $\langle \mathbb{Z}, R \rangle$ (a) without parameters and (b) (harder) with parameters?