Exam 1.

The solutions are due Wednesday March 13th before class. Either bring them to class, or send them by e-mail to my address (preferably in pdf). Let n be a natural number. $\mathbb{Q}^n$ denotes the group of all n-tuples of rational numbers with the operation of coordinatewise addition.

1. Prove that the group of all automorphisms of $\langle \mathbb{Q}^n, + \rangle$ with the composition operation is countable.

2. Prove that the collection of all subgroups of $\mathbb{Q}^n$ is a G_δ subset of the space $\mathcal{P}(\mathbb{Q}^n)$ and therefore forms a Polish space.

3. Show that two subgroups G_0, G_1 of $\mathbb{Q}^n$ are isomorphic if and only if there is an automorphism π of $\mathbb{Q}^n$ such that $\pi'' G_0 = G_1$.

4. Conclude that the equivalence relation E_n of isomorphism between subgroups of $\mathbb{Q}^n$ has all classes countable. Show that if $n < m$ then E_n is Borel reducible to E_m.