Exam 2

Due Wednesday March 15 before class.

(1) Let Γ be the theory in the language containing just the binary relational symbol ≤, and Γ says ``the relation ≤ is a linear ordering, and every element has an immediate successor in ≤''.
   (a) Write Γ down precisely in the language of first order logic.
   (b) Show that there is a sentence in the language, which cannot be decided from the axioms of Γ.

(2) Let (Z, ≤) be the structure of integers with the usual ordering on them. Note that subtraction is not in the language of the structure.
   (a) Show that the set {(x, y): x≤y and y-x≤n} is a definable set of pairs in this structure, for every natural number n.
   (b) Show that the set {(x, y, z): x≤y≤z and y-x≤z-y} is not a definable set of triples in this structure.

(3) Let (N, ▪, ≤) be the structure of natural numbers with multiplication and the usual ordering. Note that addition is not in the language of the structure. Prove:
   (a) The map f:N →N defined by $f(n)=n^2$ preserves the validity of all quantifier free formulas in the structure.
   (b) Use (a) to prove that the structure does not have elimination of quantifiers.