Exam 3.

Due Wednesday April 12 before class.

1. Show that the factorial function $f(n)=1\times 2\times \ldots \times n$ is primitive recursive.
2. Show that the function enumerating prime numbers in increasing order is primitive recursive. *Hint.* One way is to use the fact that the smallest prime larger than n is at most equal to the factorial of n plus 1, and then apply bounded search. There are different approaches too.
3. Let L be the language of first order logic with equality only. Let A be a nonempty set, viewed as a model for L. Design an algorithm for deciding whether A satisfies a given sentence of the language L, and prove that it works correctly.
4. Provide an example of a Σ_1 formula in one free variable in the language of Peano arithmetic which is not equivalent to any bounded formula. Provide a proof.