

### Exam 3.

The solutions are due Wednesday April 4th before class. Either bring them to class, or send them by e-mail to my address (preferably in pdf).

1. Prove in Peano Arithmetic that addition is commutative. The axioms are listed in the notes Chapter 6, page 6.
2. Consider the Fibonacci sequence:  $f(0) = 1$ ,  $f(1) = 1$ ,  $f(n + 2) = f(n) + f(n + 1)$ . Is this function recursive? Is it primitive recursive? Justify your reasoning.
3. Consider the following formulas in the language of Peano Arithmetic, with two variables  $x, y$ :  $\phi_0(x, y) = "x, y \text{ have no common prime divisor}"$ ,  $\phi_1(x, y) = "x = 2^y"$ ,  $\phi_2(x, y) = "x \text{ is divisible by } y"$ . Can you write them in a  $\Delta_0$  form? Can you write them in a  $\Sigma_1$  form?
4. Is there a sentence  $\theta$  such that Peano Arithmetic proves that  $\theta$  is equivalent to the provability of negation of  $\theta$ ? Can such a sentence be decidable in Peano Arithmetic?