Exam 3.

The solutions are due Wednesday April 11th before class. Either bring them to class, or send them by e-mail to my address (preferably in pdf).

1. Let P be a partial ordering and κ be a cardinal such that $|P| < \kappa$. Prove that $P \Vdash \check{\kappa}$ is a cardinal.

2. Let $\langle P, \leq \rangle$ be a poset. A set $A \subset P$ is called centered if every finite subset of A has a lower bound in P. P is called σ-centered if it is a union of countably many centered sets. Prove that a σ-centered poset is c.c.c.

3. Consider the Hechler forcing: P is the partial order of all pairs $p = \langle t_p, f_p \rangle$ such that $t_p \in \omega^{<\omega}$ and $f_p \in \omega^\omega$; the ordering is defined by $q \leq p$ if $t_p \subseteq t_q$, $\forall n\ f_p(n) \leq f_q(n)$, and $\forall n \in \mathrm{dom}(t_q \setminus t_p)\ t_q(n) > f_p(n)$. Prove that

- P is a partial ordering,
- P is σ-centered, and therefore c.c.c.
- if $G \subset P$ is a generic filter, let $g = \bigcup \{t_p : p \in G\}$. Prove that $g \in \omega^\omega$, and for every function $f \in V$, the set $\{n \in \omega : f(n) \geq g(n)\}$ is finite. (For short, g dominates every function in V modulo finite.)