

### Exam 3.

The solutions are due April 24th before class. Either bring them to class, or send them by e-mail to my address (preferably in pdf).

1. Let  $G$  be a compact subgroup of  $S_\infty$ . Show that for each number  $n \in \omega$ , the set  $\{g(n) : g \in G\}$  is finite.
2. Conclude that the product of all finite permutation groups is a universal compact subgroup of  $S_\infty$ .
3. If  $G$  is a locally compact Polish group acting continuously on a Polish space  $X$ , show that the orbit equivalence relation is  $F_\sigma$ .
4. Let  $E$  be the equivalence relation on the space  $X = (2^\omega)^\omega$  connecting points  $x_0$  and  $x_1$  if for all  $n \in \omega$ ,  $x_0(n)$  and  $x_1(n)$  are  $\mathbb{E}_0$ -related. Show that  $\mathbb{E}_1$  is not Borel reducible to  $E$ .