

### Take home exam 4

Due April 26 2pm

1. Let  $M$  be a universal Turing machine and  $c$  its notion of Kolmogorov complexity. Prove that the statement  $c(t) < n$  is a  $\Sigma_1$  formula in variables  $t, n$ . Use any coding of finite binary sequences by natural numbers.
2. Is the formula  $c(t) < n$  equivalent to a bounded formula? Justify your answer.
3. Let  $f$  be a function from natural numbers to natural numbers such that for every number  $n$ , if a sentence has fewer than  $n$  many characters and it is provable from PA, then it is provable in fewer than  $f(n)$  many steps. Show that the function  $f$  is not computable.