Exam 4.

The solutions are due Wednesday December 13th. Either leave them under my office door (456 Little Hall) or with the receptionist at 358 Little Hall, or send them by e-mail to my address (preferably in pdf).

1. Use the Ramsey theorem to show that every sequence $\langle \alpha_i : i \in \omega \rangle$ of pairwise distinct ordinals has an infinite increasing subsequence.

2. Use the Ramsey theorem to show that if $a \subset \omega$ is an infinite set of natural numbers, then there is an infinite set $b \subset a$ such that either every natural number bigger than 1 divides at most one number in b or there is a natural number bigger than 1 which divides all numbers in b. *Hint.* Consider the partition $f : [a]^2 \to 2$ defined by $f(n, m) = 0$ if there is no natural number bigger than 1 dividing both n and m.

3. Use the canonical Ramsey theorem to show the following: if a is an infinite set of points in a Euclidean space, then there is an infinite set $b \subset a$ such that the midpoints of the segments connecting pairs of points in b are all distinct.