Exam 4.

The solutions are due Wednesday May 2nd. Either send them by e-mail to my address (preferably in pdf), or leave them under the door of my office LIT456.

1. Prove that the Kolmogorov complexity of a palindrome x is $\leq |x|/2 + C$ where C is a constant not depending on x.

2. Show that the provability predicate in Peano Arithmetic satisfies a version of the $\Box$ introduction rule: whenever θ is a sentence of the language of Peano Arithmetic, then if $\mathrm{Prov}(\theta)$ holds then so does $\mathrm{Prov}(\mathrm{Prov}(\theta))$.

3. Show that the provability predicate in Peano Arithmetic satisfies a version of the $\Box$ distribution rule: whenever ϕ, ψ are sentences in the language of Peano Arithmetic, then $\mathrm{Prov}(\phi \to \psi) \to (\mathrm{Prov}(\phi) \to \mathrm{Prov}(\psi))$ holds.

4. Find a model in which both modal sentences $\phi \to \Box\neg\phi$ and $(\neg\phi) \to \Box\phi$ are satisfied (in all worlds).