Assignment 4.

1. Prove that the set $B = \{a \subset \omega : \Sigma\{\frac{1}{n+1} : n \in a\} < \infty\}$ is a Borel subset of $\mathcal{P}(\omega)$.

2. Prove that the set $A = \{a \subset \omega$: there is an infinite sequence $\langle n_i : i \in \omega\rangle$ of numbers in a such that n_i divides n_{i+1} for every $i\}$ is an analytic subset of $\mathcal{P}(\omega)$.

3. Prove that if X_0, X_1 are Polish spaces and $B_0 \subset X_0, B_1 \subset X_1$ are their Borel subsets, then $B_0 \times B_1 \subset X_0 \times X_1$ is also Borel.

4. Prove that if X_0, X_1 are Polish spaces and $A_0 \subset X_0, A_1 \subset X_1$ are their analytic subsets, then $A_0 \times A_1 \subset X_0 \times X_1$ is also analytic.

5. Suppose that X is a Polish space and A_n for $n \in \omega$ are pairwise disjoint analytic subsets of X. Prove that there are pairwise disjoint Borel subsets $B_n \subset X$ for $n \in \omega$ such that $A_n \subseteq B_n$.