Take home foundations exam 1.

Due Wednesday Feb. 8 at class meeting time. No cooperation.

1. In sentential calculus: Let A be a set of atomic sentences and Γ a theory using these atoms. Suppose that Γ is consistent in the sentential language given by the set A. Let B be a set of additional atomic sentences. Show that Γ is consistent even in the extended language using atomic sentences in both A and B. (Use completeness.)
2. A graph is a pair (V, E) where V is a set (of vertices) and E is a set of unordered pairs of vertices (edges). Consider the following property ϕ of a graph: the edges can be colored by two colors such that there is no monochromatic triangle. Show that a graph (V, E) satisfies ϕ just in case all its finite subgraphs satisfy it. (Use compactness of sentential calculus, not graph theory.)
3. Let R be a binary relational symbol. Find a model of the first order sentence ``$\forall x\ \exists y\ \forall z\ R(x, y)$ and not $R(y,z)$''. Find also a model of its negation.
4. In the natural deduction system for first order logic, show that the sentence ``$\exists x$ not $\phi$” proves the sentence ``not $\forall x\ \phi$''. Justify each line with inference rules.