

### Homework Assignment 4.

The only rule is no cooperation. Give rigorous proofs of five of the following statements:

1. The half-open interval  $(0, 1]$  is both  $F_\sigma$  and  $G_\delta$  subset of the real line.
2. The set  $\{x \in \omega^\omega : \sum_n \frac{1}{x(n)+1} < \infty\}$  is an  $F_\sigma$  subset of  $\omega^\omega$ .
3. Produce an open subset of  $\omega^\omega$  which is not closed. Can you use the universal open set to do that?
4. If  $X, Y$  are Polish spaces and  $B \subset X$  is a Borel set, then  $B \times Y$  is a Borel subset of  $X \times Y$ . (Use transfinite induction on the Borel rank of  $B$ .)
5. If  $X, Y$  are Polish spaces and  $B \subset X$  is an analytic set, then  $B \times Y$  is an analytic subset of  $X \times Y$ .