Instructor: Jindrich Zapletal. Office LIT456, e-mail zapletal@ufl.edu. Office hours MWF 9:35-10:25am at https://ufl.zoom.us/s/7401025557 (Links to an external site.)

General outline of the course. (MHF4203, MHF5207) This is one installment of a two-semester undergraduate logic sequence we run every year. It covers formal logic, formal proof systems, computability, and Goedel's incompleteness theorem. The second installment is Set Theory, to run in Spring 2021; it is essentially independent of the first installment.

The course is scheduled as 100% online. My office hours will be 100% online as well. There will be regular synchronous lectures MWF 12:50pm-1:40pm as well as some pre-recorded videos. I plan to run the synchronous lectures as more or less question and answer sessions, even though my plans will be probably adjusted during the term.

Grading. There will be three take-home exams, due on September 30, October 30, and December 14. There will also be a group presentation project. The exams are all equally weighted, and the project is worth half an exam. Attendance is worth half an exam, see below. There is no other basis for the final grade. The final grade will be calculated from the total score using a straight curve.

Project. I expect everybody in the class to participate in one of the group presentation projects. Possible topics for these projects are outlined below. Each group will have 30-50 minutes to present their work. I set aside eight periods for the presentations: Sep. 23, Sep. 25, Oct. 21, Oct. 23, Nov 13, Nov 16, Dec 7 and Dec 9. There are eight presentations in total which means that each group should have at least three members and some should have four.

Attendance. The Zoom videoconferences allow the host to collect attendance data together with the time spent in the session. I will count you as having attended if you spent at least 30 minutes in the session. There are 40 zoom sessions scheduled, I will consider you having earned full attendance credit if you attend at least 25 of them. I will report on your attendance score twice during the term in the Grades section.

Textbook. We will use a UF-produced text which is on track to be published by World Scientific in a year or so. You can find it in the Files section. It is not entirely finalized and I will edit it during the course. Any remarks and questions regarding the text are most welcome.

Content of the course. The content is divided into four areas as outlined below. I would think that the main theorem presented is Goedel's Incompleteness Theorem and my hope is that people leaving this course will understand what it says precisely, how it is used, and what are its consequences for the methodology of mathematics.

1. Proof theory

We develop the formal language of sentential logic and predicate (first order) logic. We then set up a system of formal reasoning in these logics, leading to the concept of a formal proof. All mathematics can be and usually is formalized using predicate logic. The main theorem proved in this part is Goedel's completeness theorem.

Projects for this part:

1. Boolean algebras
2. Hilbert calculus. There are many ways of setting up the formal reasoning system, we follow so-called natural deduction system which attempts to parallel usual human reasoning as closely as possible. The Hilbert calculus is an equivalent alternative which is set up so that it is easy to analyze as opposed to easy to use.
3. Modal logic. This is a system of formal reasoning quite different from predicate logic, used in certain special situations. There are in fact various types of modal logics.

1. Model theory

We develop the concept of a model of a theory in predicate logic: for example, any group is a model for the axioms of first order group theory. We will see that understanding a first order theory is more or less the same as understanding its models. As an application, we show that several natural theories are complete, i.e. they decide every sentence in their language.

Projects for this part:

IIA. Real closed fields. The real line with addition, multiplication, and the usual ordering is an example of a real closed field. Real closed fields have a natural complete set of axioms. A theorem of Tarski shows that there is a computer program deciding correctly which first order sentences are theorems of the theory of real closed fields.

IIB. Non-standard models of Peano arithmetic. The usual axioms of arithmetic of natural numbers have N as one of their models, but there are other models too, which have "infinite natural numbers" in them. How do they look and what are they useful for?

IIC. Non-standard analysis. One way to formalize the notion of infinitesimal increment in calculus is to consider a model which satisfies the same first order sentences as the reals but contains nonzero elements smaller than 2^{-n} for all natural numbers n.

We develop the notion of a computable function and computable and semi-computable set of natural numbers. There are several approaches (such as Turing machines and recursive functions) and we will show that they are equivalent. We also introduce the notion of a decidable theory and decidable structure with several examples. Our main result in this part is that the halting problem is not computable.

Projects for this part:

IIIA. Finite automatons. These are very simple machines capable of computing only primitive tasks. Still, their analysis is useful in industrial applications.

IIIB. Ackermann function. This is the usual example of a recursive function which is not primitive recursive. It is famous for its fast growth.

IIIC. Post systems. A competing way of setting up the notion of computability. Unlike Turing machines, these are formal symbol manipulation systems.

IIID. Hilbert's 10^{th} problem. One of the famous list of problems from the first international congress of mathematicians in 1900 asks for an algorithm to decide whether a given polynomial with integer coefficients has integer roots. In 1970, it was shown that there is no such an algorithm.

1. Peano Arithmetic and Goedel's Incompleteness Theorem.

This part contains the main result of this course. Namely, the usual axiomatization of the arithmetic of natural numbers is not complete; i.e. there are arithmetical yes-no questions to which it does not give an answer. In fact, no reasonable axiomatization can be complete. This leaves us in famously uncomfortable philosophical situation.

Projects for this part:

IVA. Pressburger arithmetic. If we drop multiplication, the resulting theory of addition of natural numbers is much simpler.

IVB. Modal logic of provability. The most common proof of Goedel's incompleteness theorem formalizes the notion of a proof and provability as arithmetic statements. The resulting formulas are famously confusing. However, there is a modal logic completely characterizing the behavior of provability, and it compresses even the most convoluted arguments about provability into a couple of lines.

Further administrative matters.

Requirements for class attendance and make-up exams, assignments, and other work in this course are consistent with university policies that can be found in the online catalog (Links to an external site.) (Links to an external site.).

Students are expected to provide professional and respectful feedback on the quality of instruction in this course by completing course evaluations online via GatorEvals. Guidance on how to give feedback in a professional and respectful manner is available here (Links to an external site.) (Links to an external site.). Students will be notified when the evaluation period opens, and can complete evaluations through the email they receive from GatorEvals, in their Canvas course menu under GatorEvals, or via this link (Links to an external site.) (Links to an external site.). Summaries of course evaluation results are available to students here (Links to an external site.) (Links to an external site.).

Students with disabilities requesting accommodations should first register with the UF Disability Resource Center (352.392.8565) by providing appropriate documentation. Once registered, students will receive an accommodation letter which must be presented to the instructor when requesting accommodations. Students with disabilities should follow this procedure as early as possible in the semester.

The Mathematics Department is committed to diversity and inclusion of all students. We acknowledge, respect, and value the diverse nature, background and perspective of students and believe that it furthers academic achievements. It is our intent to present materials and activities that are respectful of diversity: race, color, creed, gender, gender identity, sexual orientation, age, religious status, national origin, ethnicity, disability, socioeconomic status, and any other distinguishing qualities.

The UF Religious Holidays Policy is available here. (Links to an external site.) (Links to an external site.)

UF students are bound by The Honor Pledge which states, “We, the members of the University of Florida community, pledge to hold ourselves and our peers to the highest standards of honor and integrity by abiding by the Honor Code.” On all work submitted for credit by students at the University of Florida, the following pledge is either required or implied: “On my honor, I have neither given nor received unauthorized aid in doing this assignment.” The Honor Code (Links to an external site.) (Links to an external site.) specifies a number of behaviors that are in violation of this code and the possible sanctions. Furthermore,you are obligated to report any condition that facilitates academic misconduct to appropriate personnel. If you have any questions or concerns, please consult with the instructor or TAs in this class.

Our class sessions may be audio-visually recorded for students in the class to refer back and for enrolled students who are unable to attend live. Students who participate with their camera engaged or utilize a profile image are agreeing to have their video or image recorded. If you are unwilling to consent to have your profile or video image recorded, be sure to keep your camera off and do not use a profile image. Likewise, students who un-mute during class and participate orally are agreeing to have their voice recorded. If you are not willing to consent to have your voice recorded during class, you will need to keep your mute button activated and communicate exclusively using the "chat" feature, which allows students to type questions and comments live. The chat will not be recorded or shared. As in all courses, unauthorized recording and unauthorized sharing of recorded materials by students or any other party is prohibited.