Speaker: Andrew Marks

Title: A constructive solution to Tarski's circle squaring problem

Abstract: In 1925, Tarski posed the problem of whether a disc in R^2 can be partitioned into finitely many pieces which can be rearranged by isometries to form a square of the same area. Unlike the Banach-Tarski paradox in R^3, it can be shown that two Lebesgue measurable sets in R^2 cannot be equidecomposed by isometries unless they have the same measure. Hence, the disk and square must necessarily be of the same area.

In 1990, Laczkovich showed that Tarski's circle squaring problem has a positive answer using the axiom of choice. We give a completely constructive solution to the problem and describe an explicit (Borel) way to equidecompose a circle and a square. This answers a question of Wagon.

Our proof has three main ingredients. The first is work of Laczkovich in Diophantine approximation. The second is recent progress in a research program in descriptive set theory to understand how the complexity of a countable group is related to the Borel cardinality of the equivalence relations generated by its Borel actions. The third ingredient is ideas coming from the study of flows in networks.

This is joint work with Spencer Unger.