Dates: May 11 and 18, 2021

Speaker: Jonathan Cancino Manriquez

Title: Every maximal ideal may be Katetov above of all F_σ ideals

Abstract: We will define the Katetov order and the notion of I-ultrafilters, both of which are very useful in the classification of combinatorial properties of ultrafilters. Then we will review some structural aspects of Katetov order, and introduce a question from M. Hrusak and O. Guzman, asking about the existence of I-ultrafilters when I is an F_σ ideal. Afterwards we proceed to answer this question, proving the consistency of the assertion of the title of this talk. This result has particular interest when we turn our attention to Hausdorff ultrafilters. Hausdorff ultrafilters are defined as those ultrafilters for which the ultrapower of ω modulo the ultrafilter, when considering the topology induced by the discrete topology from ω, is a Hausdorff topological space. M. Hrusak and D. Meza Alcantara have proved that Hausdorff ultrafilters are exactly the G_{fc}-ultrafilters, where G_{fc} is the ideal on $[\omega]^2$ of graphs with finite chromatic number, which is an F_σ ideal, so our result implies a solution to the question about the existence of Hausdorff ultrafilters, that is, the consistency of the nonexistence of Hausdorff ultrafilters.