All Borel homomorphisms between Polish groups are continuous. For some Polish groups, discontinuous homomorphisms may be produced using the axiom of choice, for others there are no discontinuous homomorphisms of any type regardless of the status of the axiom of choice. In this talk, we discuss several independence results between existence of discontinuous homomorphisms and othe types of "paradoxical" objects in choiceless set theory. The results are from the book "Geometric Set Theory", jointly co-authored with Paul Larson, in print with AMS Surveys and Monographs.