Assignment 2.

1. Suppose that $\langle P, \leq_P \rangle$ and $\langle Q, \leq Q \rangle$ are two well-ordered sets. Define an ordering on $P \times Q$ by letting $\langle p_0, q_0 \rangle \leq \langle p_1, q_1 \rangle$ if either $p_0 <_P p_1$ or $p_0 = p_1$ and $q_0 \leq_Q q_1$. Prove that $\leq$ is a well-ordering.

2. Prove that for every ordinal α, $\alpha \in V_{\alpha+1}$.

3. Prove that for every ordinal α, $\alpha \notin V_\alpha$.

4. Suppose that x, y are sets such that $|x| < |y|$. Prove that every partial injection from x to y can be extended to a total injection of x to y, i.e. one whose domain is the whole set x. What happens if we allow $|x| = |y|$?

5. Let $\langle P, \leq \rangle$ be a partially ordered set. Prove that there is a set $A \subset P$ such that no two distinct elements of A are $\leq$-comparable and every element of P is $\leq$-comparable with some element of A.