Assignment 2.

1. In the theory of dense linear order without endpoints, find a quantifier-free equivalent to the following formula $\phi(x, y)$: $\forall u \exists v\ v \leq u \wedge (x \leq u \to y \leq v)$.

In the following two problems, consider the structure $\langle \mathbb{Z}, +, 0 \rangle$ of integers with addition.

2. Show that every subset of $\mathbb{Z}$ definable by a quantifier-free formula is either finite or its complement is finite.
3. Use the result of the previous item to find a subset of $\mathbb{Z}$ which is definable in the structure, but not definable with a quantifier-free formula. Conclude that the theory of the structure does not have quantifier elimination.

In the last two problems, consider the structure $\langle \mathbb{Q}, +, 0 \rangle$ of rational numbers with addition.

4. Prove that the theory of the structure has quantifier elimination.
5. Use the previous item to show that the function $f(x) = x^2$ is not definable in the structure. *Hint.* Functions definable by quantifier-free formulas must be linear. (Prove the hint.)