Homework Assignment 3.

The only rule is no cooperation. Give rigorous proofs of the following statements:

1. Let $\mathbb{Q} = A_0 \cup B_0$ be a partition of rationals into two dense sets. Let $\mathbb{Q} = A_1 \cup B_1$ be another partition of rationals into two dense sets. Prove that there is an order-preserving bijection $h : \mathbb{Q} \to \mathbb{Q}$ such that $h'' A_0 = A_1$ and $h'' B_0 = B_1$. (Use a back-and-forth construction to obtain h.)

2. Let X, Y be topological spaces. Let $f : X \times Y \to X$ be the projection function defined by $h(x, y) = x$. Prove that f is continuous. (Here, $X \times Y$ is equipped with the product topology.)

3. Let X be a set and d, e two metrics on it. Prove that the function f given by $f(x, y) = \min(d(x, y), e(x, y))$ is again a metric on X.

4. The Cantor space is homeomorphic to a closed subset of the Baire space.

5. Let $\langle X, T\rangle$ and $\langle Y, S\rangle$ be two Polish spaces with $X \cap Y = 0$. Let U be the topology on the set $X \cup Y$ generated by $T \cup S$. Prove that $\langle X \cup Y, U\rangle$ is a Polish space.