Assignment 3.

$\mathfrak{N}$ denotes the natural numbers with addition, multiplication, ordering, and the successor function.

1. Let ϕ be Gödel's undecidable sentence (so ϕ is equivalent to "ϕ is not provable from the axioms of Peano Arithmetic"). Prove that $\mathfrak{N} \models \phi$. How is it possible that we can prove that ϕ holds in $\mathfrak{N}$ when we know that ϕ is not provable from PA?

2. Let ψ be any sentence true in $\mathfrak{N}$. Show that there is a sentence ϕ which is not decidable in the theory PA+ψ.