Assignment 3.

Due Wednesday Nov. 19 before class.

1. Show that every Polish space is either countable or has the same cardinality as $\mathcal{P}(\omega)$.

2. Show that a product of countably many Polish spaces is separable. (I skipped this part when I proved that the product is Polish.)

3. Suppose that $d$ is a metric on a space $X$. Show that $\sqrt{d}$, the function given by $\sqrt{d}(x, y) = \sqrt{d(x, y)}$, is a metric as well. Can you prove a more general theorem?

4. Show that the metrics $d$ and $\sqrt{d}$ generate the same topology.

5. Is there a subset (not necessarily closed) of $2^{\omega}$ which is homeomorphic to $\omega^{\omega}$? Why?