

Assignment 3.

Due Wednesday Nov. 19 before class.

1. Show that every Polish space is either countable or has the same cardinality as $\mathcal{P}(\omega)$.
2. Show that a product of countably many Polish spaces is separable. (I skipped this part when I proved that the product is Polish.)
3. Suppose that d is a metric on a space X . Show that \sqrt{d} , the function given by $\sqrt{d}(x, y) = \sqrt{d(x, y)}$, is a metric as well. Can you prove a more general theorem?
4. Show that the metrics d and \sqrt{d} generate the same topology.
5. Is there a subset (not necessarily closed) of 2^ω which is homeomorphic to ω^ω ? Why?