Abstract: A reduction from one equivalence relation $E$ to another one $F$ is a function $f$ for which $x E y$ iff $f(x) F f(y)$. If $f$ is a Borel function, we call it a Borel reduction; while, if $f$ is computable, it is a computable reduction. (The fields of $E$ and $F$ must be appropriate to these classes of functions.) Computable reducibility has been the subject of recent investigations by a number of researchers, spurred on by the analogy to Borel reducibility, which has been a standard topic for several decades.

We will discuss joint work with Keng Meng Ng, examining a weaker version of computable reducibility. An $n$-reduction from $E$ to $F$ is a function $f$ which, on input $\langle x_1, \ldots, x_n \rangle$, outputs $\langle y_1, \ldots, y_n \rangle$ such that $x_i E x_j$ iff $y_i F y_j$, and a finitary reduction does the same uniformly for all $n$. Computable finitary reducibility turns out to have interesting properties. For example, it is known from work of Ianovski, Nies, and the two of us that there is no $\Pi^0_2$ equivalence relation which is complete among all $\Pi^0_2$ equivalence relations under computable reducibility. However, under computable finitary reducibility, the $\Pi^0_2$ relation $i E j$ iff $W_i = W_j$ is complete in this sense, and this generalizes nicely to $\Pi^0_m$ for all $m > 2$. Additionally, for every Turing degree $d$, there exist equivalence relations $E$ and $F$ on $\omega$ such that $E$ is computably finitarily reducible to $F$, but there is no $d$-computable reduction from $E$ to $F$. Finally, we will give a natural $\Pi^0_3$ equivalence relation which is $\Pi^0_3$-complete for computable 3-ary reducibility, but not for computable 4-ary reducibility. It remains open to what extent similar results hold for finitary Borel reducibility, and, since Borel reductions deal with equivalence relations on $2^\omega$, one might also inquire into countable Borel reducibility.