

Speaker: Jason Rute

Title: Randomness, Brownian Motion, Riesz Capacity, and Complexity

Abstract: *Algorithmic randomness* is a topic in computability theory which investigates which paths in a stochastic process behave randomly (with respect to all computable statistical tests). *Riesz capacity* is an important concept in potential theory and stochastic processes. For example, it is used to estimate the probability that a Brownian motion is zero on a given set of times. *A priori complexity* $KM(x)$ is a measure of the computational complexity of a finite bit string x . The following results connects these subjects.

For all t in $(0, 1]$, the following are equivalent:

1. t is a zero of some Martin-Löf random one-dimensional Brownian motion.
2. t is Martin-Löf random with respect to $1/2$ -Riesz capacity.
3. $\sum_n 2^{n/2 - KM(t[0, n-1])} < \infty$ where $t[0, n-1]$ is the first n bits of the binary expansion of t .

The equivalence of (2) and (3) is joint work with Joseph Miller.

This is part of a broader program exploring randomness for capacities. Capacity theory provides a unified framework to study a number of topics in algorithmic randomness—including strong s -randomness, s -energy randomness, algorithmically random closed sets, effective Hausdorff dimension, randomness for classes of measures, and randomness for semimeasures. Moreover, capacity theory—which has been thoroughly investigated over the last 60 years—provides a wealth of classical results to draw upon to prove new results in algorithmic randomness.