

**Date:** September 29

**Speaker:** Thomas Winckler

**Title:** A Countable Antichain Theorem for finite functions is ZF-equivalent to "The Countable Union of Finite Sets is Countable".

**Abstract:** Let  $X$  be any set. Let  $P$  equal the finite partial functions from  $X$  into the natural numbers, where  $P$  is partially ordered by inclusion. A subset of  $P$ , say  $Q$ , is an antichain iff the finite functions in  $Q$  are pairwise incompatible. Let  $p$  and  $q$  be two functions.  $p$  and  $q$  are incompatible iff there exists some  $x$  such that  $p(x)$  does not equal  $q(x)$ . We'll prove, (just using ZF, and with no fragment of the axiom of choice), that "all subsets of  $P$  (that are antichains), must be countable" iff "the countable union of finite sets is countable". Proofs in the literature prove the Countable Antichain Theorem, by assuming "the countable union of countable sets is countable".